FINAL PROJECT

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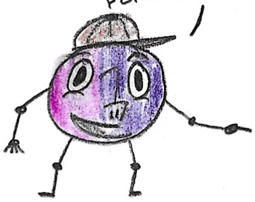
TOPIC ①

BASIC ALGEBRA

- Algebra is and will continue to be essential in any math courses you take. Having knowledge like order of operation (PEMDAS) and basic mathmatic properties (assosciative, commutative, identity, etc.)
- If you take each problem step by step using basic algebra strategies, it makes all math much easier

Parenthesis
Exponent
Multiply
Divide
Add
Subtract

left to right



Ex: Solve for X

① $\dfrac{7x + (8-3)}{2 \cdot 3 + 6} = 24$

$\dfrac{7x + 5}{6 + 6} = 24$

$12 \cdot \dfrac{7x + 5}{12} = 24 \cdot 12$

$7x + 5 = 288$
$\quad\;\; -5 \quad\;\; -5$

$\dfrac{7x = 283}{7}$

$x = \dfrac{283}{7}$

TOPIC ②

VALUE MANIPULATION

- Sometimes, values like fractions can seem a little daunting. Knowing how to manipulate and simplify complex values is another strategy that makes all math easier, not just precalculus
- Most people love decimals, but trust me, fractions are actually a whole lot simpler.

Ex: Simplify

① $\dfrac{2401}{7} \div \dfrac{7}{7} = \dfrac{343}{1}$ OR 343

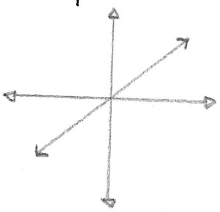
② $\dfrac{2/3}{6/5} \rightarrow \dfrac{2}{3} \cdot \dfrac{5}{6} = \dfrac{10}{18} \div \dfrac{2}{2} = \dfrac{5}{9}$

③ $\dfrac{\cancel{256}}{\cancel{17}} \cdot \dfrac{\cancel{17}}{\cancel{256}} = 1$

$\dfrac{0.\overline{666}}{1.2} \rightarrow 0.67 \div 1.2 \approx 0.558\overline{333}$ ← NOT COOL!

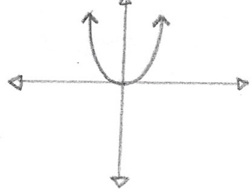
PARENT FUNCTIONS & GRAPHING

$y = x$



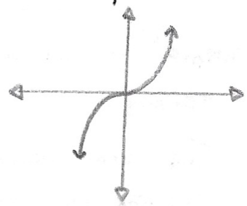
LINEAR

$y = x^2$



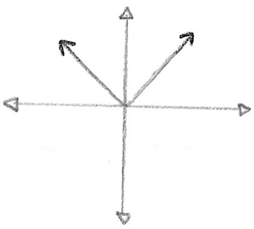
QUADRATIC

$y = x^3$



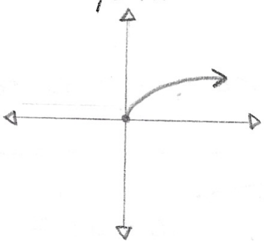
CUBIC

$y = |x|$



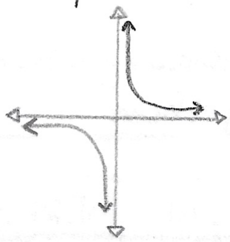
ABSOLUTE VALUE

$y = \sqrt{x}$



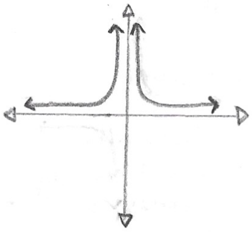
RADICAL

$y = \frac{1}{x}$



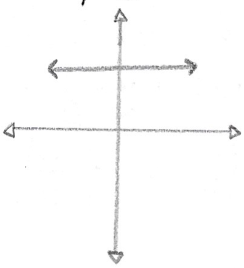
RATIONAL

$y = \frac{1}{x^2}$



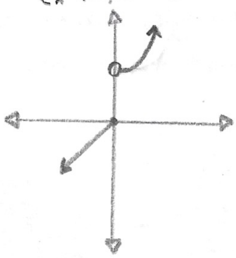
RATIONAL SQUARED

$y = c$



CONSTANT

$y = \begin{cases} x, & x \leq 0 \\ x^2+1, & x > 0 \end{cases}$



PIECE-WISE

- Knowing these will definitely help when you begin to graph in pre-calc

REMEMBER: $A \cdot f(Bx + C) + D = y$

vertical stretch/compression; horizontal stretch/compression; horizontal shift; vertical shift

- If you have multiple transformations in an equation, it follows this order:

① horizontal shifts

② horizontal & vertical stretch/compressions

③ reflections

④ vertical shifts

- Point-plotting is the most sure-fire way to find a graph's points

x	0	1	2	3
y	1	2	3	4

→ Coordinates

(0,1) (2,3)
(1,2) (3,4)

It doesn't always look this complicated. It can look like this, too.

$f(Bx) = y$

This equation only involves a horizontal stretch/compression.

- Translations and such come up often when graphing. By memorizing the parent functions, you can break down a graphing problem to figure out the graph's movement.

POLYNOMIALS & BINOMIAL THEREOM

• A polynomial is an expression with multiple terms

- The term "polynomial" covers all expressions, like binomial or monomial

EXAMPLE: $\underbrace{2x + 4}_{\text{two terms}}$ = binomial

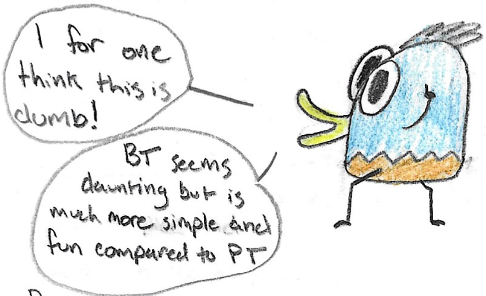
*Any expression under the sun with one or more terms can be referred to as a polynomial

• Just knowing what polynomials are will help reduce the stress when seeing them

BINOMIAL THEREOM

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} \cdot b^k$$

• Most people are taught Pascal's Triangle, which is said to be simpler than the binomial thereom



• Pascal's Triangle helps with only a portion of a binomial expansion and requires many more steps:

1
1 + 1
1 + 2 + 1
1 + 3 + 3 + 1
1 + 4 + 6 + 4 + 1
1 + 5 + 10 + 10 + 5 + 1
1 + 6 + 15 + 20 + 15 + 6 + 1

*the midpoint is where values start to repeat themselves

check it out here too!

Tells base leading coefficients

not scary → $\sum$ = this means that all the terms we get will be added together

$k=0$ → reminds us that our sequence will start at 0

$\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ •This deals with factorials which take a number and multiply it down until it reaches 1

Ex: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

EXAMPLE:

$(x+4)^5 = \sum_{k=0}^{5} \binom{5}{k} x^{5-k} \cdot 4^k$

*the k value will start at zero and rise until it reaches the n value

*let's start with the $\binom{n}{k}$

$\binom{5}{0} = 1 \cdot x^5 \cdot 1$ → these values came from this

$\binom{5}{1} = \dfrac{120}{1!(4!)} = 5 \cdot x^4 \cdot 4$

$\binom{5}{2} = \dfrac{120}{2!(3!)} = 10 \cdot x^3 \cdot 16$

$\binom{5}{3} = \dfrac{120}{3!(2!)} = 10 \cdot x^2 \cdot 64$

$\binom{5}{4} = \dfrac{120}{4!(1!)} = 5 \cdot x^1 \cdot 256$

$\binom{5}{5} = 1 \cdot x^0 \cdot 1024$

*the first and last value will always be 1

Take these terms, simplify them, and then add it all together

1st term $= x^5$
2nd term $= 20x^4$
3rd term $= 160x^3$
4th term $= 640x^4$
5th term $= 1280x$
6th term $= 1024$

•All of this work tells us the final expansion of $(x+4)^5$

$$(x+4)^5 = \underline{x^5 + 20x^4 + 160x^3 + 640x^4 + 1280x + 1024}$$

LOGARITHMS

LOGARITHMIC EXPRESSIONS

- logarithms (logs) trip people up more than they should
- with logs it is handy to know how to deal with exponents as well as a handful of "rules"
- Knowing logs will help cover a large portion of pre-calc

- The rules of logarithms are as follows:

Product Rule

$\log_x(yz) = \log_b y + \log_b z$

Quotient Rule

$\log_x\left(\frac{y}{z}\right) = \log_x y - \log_x z$

Power Rule

$\log_x y^z = z \cdot \log_x y$

REMEMBER THESE ESPECIALLY

- The basics of logarithms are as follows:

$\log_{x} y = z$

base · solution · exponent

$\log_x y = z \longrightarrow x^z = y$

- If a logarithm has nothing in the base spot, the base is 10. This is called a common log ($\log x = \log_{10} x$)

- There is something called a NATURAL LOG which deals with a base of "e"

  - e is like π, it is a symbol assigned to a specific value ($e \approx 2.71828$)

- A natural log equation looks like this:

$\ln_e x = y$

Natural logs could easily be written as:

$\log_e x = y$

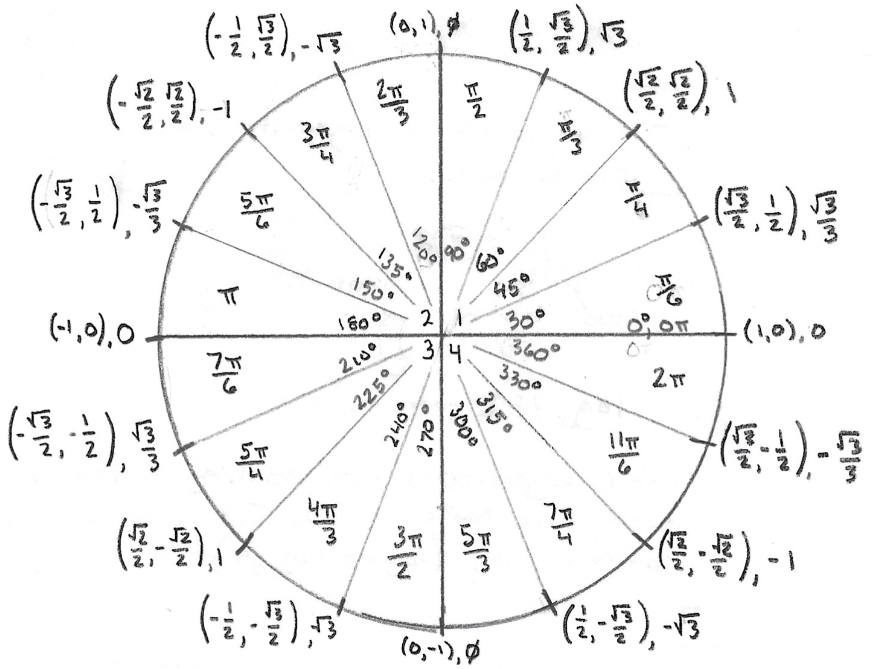
But the "ln" acts like a notice to the mathmetician that the base is e

* It should be noted that e is just another base, so the same logarithmic rules apply

THE UNIT CIRCLE AND IT'S INVERSE

NORMAL

Key: (cos, sin), tan

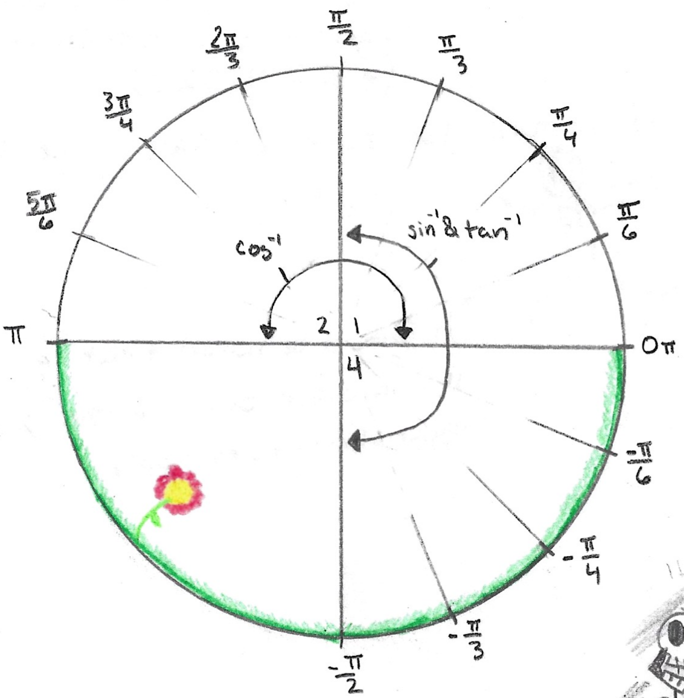


- I cannot stress how important both of these circles are. The unit circle is used constantly in pre-calc so get used to it.

- At first, knowing the degrees and radians is enough, but eventually it becomes essential to know the cos, sin, and tan too.

INVERSE

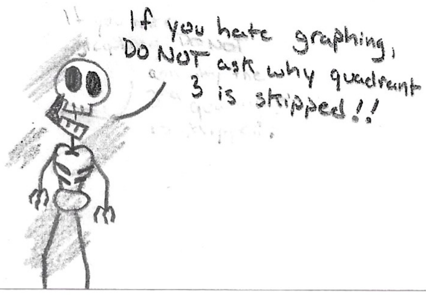
*The degrees, cos, sin, and tan are the same as the normal unit circle



- The inverse unit circle is used for inverse trig. functions:

$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
arcsin x	arccos x	arctan x

- To avoid a mixup when working with tangents, there are domain restrictions.
  - $\cos^{-1}$ only functions in quadrants one and two
  - $\sin^{-1}$ and $\tan^{-1}$ only function in quadrants one and four



TOPIC 10

TRIGONOMETRIC IDENTITIES

- Trig. identities introduces the use of csc, sec, and cot into pre-calc
- Each identity is there to help you solve or simplify trig. equations, so it is important that you learn these

EXAMPLE: Simplify

① $\dfrac{\csc \cdot \tan}{\csc^2 - \cot^2} \rightarrow \dfrac{1}{\sin} \cdot \dfrac{\sin}{\cos} = \sec$

$1 + \cot^2 = \csc^2$
$-\cot^2 \quad -\cot^2$

$1 = \csc^2 - \cot^2$

$\sec =$

Reciprocal Identities:

$\csc = \dfrac{1}{\sin}$	$\sec = \dfrac{1}{\cos}$	$\cot = \dfrac{1}{\tan}$

*Working with cos and sin is ideal because they cannot be simplified further

Quotient Identities:

$\tan = \dfrac{\sin}{\cos}$	$\cot = \dfrac{\cos}{\sin}$

*cot is alphabetical top to bottom, tan isn't

Pythagorean Identities:

$\sin^2 + \cos^2 = 1$	$1 + \cot^2 = \csc^2$	$\tan^2 + 1 = \sec^2$

*this identity can be manipulated to obtain the other two

REFLECTION ESSAY

Hey there, new pre-calc students. What a class you've entered. I'm sure some of you are dreading another year of math, but it seriously won't be absolutely horrible. I will admit that there are hard concepts in Mr. Brenneman's class, but that is no reason to quit because you can still succeed overall. Now, if math is your forte, good job, you're only halfway to succeeding. Here are some tips to help both parties prepare for the coming year. First off, class rules. Brenneman has these rules posted up and if the class follows them, you get extra credit. It is the simplest way to earn some. But, of course, Brenneman has other ways to get some sweet EC. Homework is graded on completion and if you do all the homework, that's extra credit. No math skills needed, but it sure does help. You can even do test retakes and your final project is the biggest cake walk ever. But disregarding all of that, the biggest tip I can give you is don't go into this class having already given up. In all aspects, that is the stupidest mindset and frankly it annoys me. You have to try to even hope for a good grade, so don't give up too early. For those good at math, don't rub it in, but help those who don't want to try. It will make both of your experiences in pre-calculus so much better.