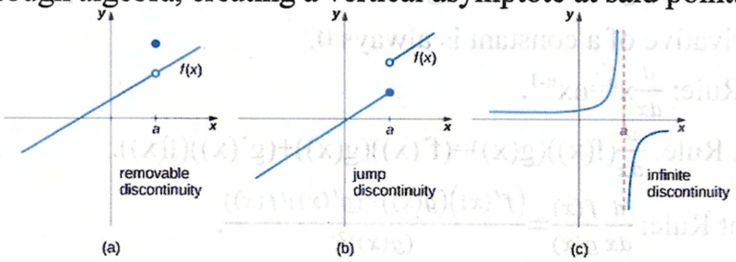
1. Limits and Continuity

Old Stuff:

- Removable Discontinuities: There is a hole in the graph that can be filled, usually by factoring out terms algebraically.
- Jump Discontinuities: The function jumps from one value to another at a point called c. At point c the limit as x approaches c from the left is unequal to the limit as x approaches c from the right.
- Infinite Discontinuities: The denominator equals 0 at a point and this 0 can't be removed through algebra, creating a vertical asymptote at said point.



- When evaluating limits we first use substitution, if this fails and we get k/0 we usually try to use algebraic manipulation (factoring, conjugate pairs, etc.) to remove the discontinuity and evaluate the limit. However, if we get 0/0 or ∞/∞ and all else fails we use L'Hopital's Rule to evaluate the limit.
- L'Hopital's Rule: If $\lim_{x \to c} f(x) = 0/0$ or ∞/∞ then take the derivative of both the numerator and denominator and try to use substitution again, you repeat this process until you don't get 0/0 or ∞/∞.
- If $\lim_{x \to c-} f(x) \neq \lim_{x \to c+} f(x)$ then $\lim_{x \to c} f(x)$ does not exist.
- If f(x) is differentiable it must also be continuous.

New Stuff:

There is no new information to learn in this section for AP BC Calculus, however it is important to make sure that you have mastered limits as they are used for many new BC Calc concepts including the integration of improper integrals, the nth term test, the integral test, the limit comparison test, the alternating series test, and the ratio test.

AP Exam:

From my experience I would say that limits and continuity make up around 5%-10% of the AP exam, mainly the no calculator MCQ portion. They also tend to mix these concepts in with other concepts like those mentioned in the section above.

2. Differentiation

Old Stuff:

- Limit Definition of the Derivative: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
- The derivative is the instantaneous rate of change of a function at a specific point.
- Slope formula ($\frac{y_1-y_2}{x_1-x_2}$)gives us the average rate of change over an interval.
- MVT: If f(x) is continuous and differentiable over an interval then f'(x) must equal the average rate of change over the interval at some point c.
- The derivative of a constant is always 0.
- Power Rule: $\frac{d}{dx}x^n=nx^{n-1}$.
- Product Rule: $\frac{d}{dx}(f(x))(g(x))=(f'(x))(g(x))+(g'(x))(f(x))$.
- Quotient Rule: $\frac{d}{dx}\frac{f(x)}{g(x)}=\frac{(f'(x))(g(x))-(g'(x))(f(x))}{(g(x))^2}$.
- Chain Rule: $\frac{d}{dx}f(g(x))=(f'(g(x)))(g'(x))$.
- Trig Derivatives: $\frac{d}{dx}\sin(x)=\cos(x)$, $\frac{d}{dx}\cos(x)=-\sin(x)$, $\frac{d}{dx}\sec(x)=\sec(x)\tan(x)$, $\frac{d}{dx}\csc(x)=-\csc(x)\cot(x)$, $\frac{d}{dx}\tan(x)=\sec^2(x)$, $\frac{d}{dx}\cot(x)=-\csc^2(x)$.
- Arc-Trig Derivatives: $\frac{d}{dx}\sin^{-1}(x)=\frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\cos^{-1}(x)=\frac{-1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\sec^{-1}(x)=\frac{1}{|x|\sqrt{x^2-1}}$, $\frac{d}{dx}\csc^{-1}(x)=\frac{-1}{|x|\sqrt{x^2-1}}$, $\frac{d}{dx}\tan^{-1}(x)=\frac{1}{x^2+1}$, $\frac{d}{dx}\cot^{-1}(x)=\frac{-1}{x^2+1}$.
- $\frac{d}{dx}e^x=e^x$ and $\frac{d}{dx}\ln(x)=1/x$.
- Inverse Functions: $\frac{d}{dx}f^{-1}(x)=\frac{1}{f'(f^{-1}(x))}$.
- Implicit Differentiation: Differentiate both sides and use chain rule on the y's.

New Stuff:

- $\frac{d}{dx}a^x=\ln(a)(a^x)$ and $\frac{d}{dx}\log_a x=\frac{d}{dx}\frac{\ln(x)}{\ln(a)}=\frac{1}{x\ln(a)}$.

AP Exam:

A large part of the AP Calculus BC Exam is differentiation, typically they'll take trig or arc-trig functions and mix them in with rules like chain rule or product rule. As for the new stuff I never saw a question on differentiating those, so they're not crucial to learn. Additionally, I only had 1 question involving differentiating inverse functions, so it may not be worth your time memorizing the formula.

3. Applications of Differentiation

Old Stuff:

- Related Rates: Find a general formula for what you're representing, figure out which values are constants and which are not, use implicit differentiation to differentiate both sides, and plug in your values.
- First Derivative Test: f(x) has critical points where f'(x)=0 or DNE. If f'(x) changes from positive to negative f(x) has a relative max at this point, if f'(x) changes from negative to positive f(x) has a relative min at this point, and if f'(x) doesn't change signs it's not a relative extrema.
- Second Derivative Test: If f'(x)=0 and f''(x)=+ then f(x) has a relative min at this point, if f'(x)=0 and f''(x)=- then f(x) has a relative max at this point, and if f'(x)=0 and f''(x)=0 then f(x) may have a point of inflection at this point.
- Candidates Test: Test all critical points and endpoints to find the absolute extrema.
- If f''(x)=+ then f(x) is concave up at this point, if f''(x)=- then f(x) is concave down at this point, and if f''(x)=0 then f(x) may have a point of inflection at this point. Points of inflection are where f''(x) changes signs.
- If f'(x)=+ then f(x) is increasing at this point, of f'(x)=- then f(x) is decreasing at this point, and if f'(x)=0 then f(x) may have relative extrema at this point. Relative extrema are where f'(x) changes sign.
- In order to sketch a curve carry out all the tests listed above to learn about the curve's concavity and where it's increasing and decreasing.
- Linear Motion: When f'(x) and f''(x) have different signs the particle is slowing down, when f'(x) and f''(x) have the same sign the particle is speeding up, when f'(x)=0 the particle is stopped, and when f''(x)=0 the particle is moving at a constant speed.

New Stuff:

None, however you will gain a more in depth understanding of particle motion when you learn about vector-valued functions and parametric equations.

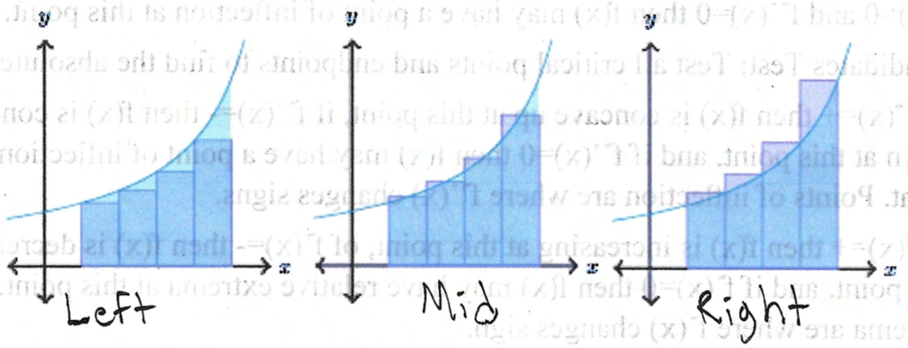
AP Exam:

On the AP BC Calculus Exam you will likely have to analyze an f' graph on the FRQ section. Additionally, during the MCQ section there were a few questions where we had to determine relative extrema, intervals of concavity, and intervals where f was increasing or decreasing. We were also given a related rates FRQ.

4. Integration

Old Stuff:

- The integral is the area under the curve.
- FTOC: The integral of a function is its antiderivative, therefore $\frac{d}{dx}\int_a^x f(t)dt$=f(x).
- FTOC II: $\int_a^x f(t)dt$=F(x)-F(a).
- Riemann Sums: Left Riemann sum (left to right, rectangle area), Right Riemann sum (right to left, rectangle area), trapezoidal sum (take both sides of subintervals as $y_1=b_1$ and $y_2=b_2$ and the distance between x_1 and x_2 a, trapezoid area $\frac{h(b_1+b_2)}{2}$), and midpoint sum (use the y value of the point right between x_1 and x_2, rectangle area).



- Reverse Power Rule: $\int x^n dx = \frac{x^{n+1}}{n+1}$+C. ALWAYS REMEMBER YOUR +C.
- When integrating remember that the integral is the antiderivative, so all you have to do is figure out what you have to take the derivative of to get the value inside the integral. For example, $\int \sin(x)\, dx$=-cos(x)+C or $\int \frac{1}{x} dx$=ln|x|+C.
- U Sub: $\int \frac{2x}{x^2+2}$dx, u=x^2+2, $\frac{du}{dx}$=2x, dx=$\frac{du}{2x}$, $\int \frac{1}{u}$du=ln$|u|$+c=ln$|x^2+2|$+C.

New Stuff:

- Integration Using Partial Fraction Decomposition: $\int \frac{2x}{(x+1)(2x+1)}$dx=$\int \frac{A}{x+1} + \frac{B}{2x+1} dx$, 2x=2Ax+A+Bx+B, A+B=0, A=-B, 2A+B=2, -2B+B=2, -B=2, B=-2, A=2, $\int \frac{2x}{(x+1)(2x+1)}$dx=$\int \frac{2}{x+1} - \frac{2}{2x+1} dx$= 2ln$|x+1|$-ln$|2x+1|$+C.

- Improper Integrals: An integral is improper if it has a discontinuity within its bounds or if one of its bound is ∞ or -∞. For example, $\int_0^{\infty} \frac{-1}{(x+1)^2}dx = \lim_{b\to\infty} \int_0^b \frac{-1}{(x+1)^2}dx = \lim_{b\to\infty} \left[\frac{1}{x+1}\right]_0^b = \lim_{b\to\infty} \frac{1}{b+1} - 1 = 0 - 1 = -1$.
- Integration by Parts: Use LIATE (log, inverse, algebra, trig, exponential) to determine u. Then use $\int udv = uv - \int vdu$.
- Arc-Trig Integration: $\int \frac{1}{a^2+u^2} dx = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$, $\int \frac{1}{\sqrt{a^2-u^2}} dx = \sin^{-1}(\frac{u}{a}) + C$, $\int \frac{1}{u\sqrt{u^2-a^2}} dx = \frac{1}{a}\sec^{-1}(\frac{|u|}{a}) + C$.

AP Exam:

When I took the AP exam there was only 1 arc-trig integration question, however there were probably 5-6 MCQ questions involving integration by parts, improper integrals, or partial fraction decomposition. We also had a Riemann sum FRQ. It's safe to say that you should master integration considering it's a big chunk of calculus.

5. Applications of Integration

Old Stuff:

- Average Value of a Function: $\frac{1}{b-a}\int_a^b f(x)\,dx$.
- Distance: $\int_a^b |v(t)|dt$; Displacement: $\int_a^b v(t)dt$.
- Area Between Two Curves: If one curve is on top of the other use $\int_a^b y_1 - y_2 dx$, in this case the upper function is y_1, the lower function is y_2, and the bounds are x-values. If one curve is to the right of the other use $\int_a^b x_1 - x_2 dy$, in this case the rightmost curve is x_1, the leftmost curve is x_2, and the bounds are y-values.
- Volume Using Disc Method: $\pi\int_a^b (R(x))^2 dx$ when rotating around the x-axis or lines parallel to the x-axis and $\pi\int_a^b (R(y))^2 dy$ when rotating around the y-axis or lines parallel to the y-axis.
- Volume Using Washer Method: $\pi\int_a^b (R(x))^2 dx$- $\pi\int_a^b (r(x))^2 dx$ when rotating around the x-axis or lines parallel to the x-axis, R(x) is the larger/outer radius and r(x) is the smaller/inner radius. $\pi\int_a^b (R(y))^2 dy$- $\pi\int_a^b (r(y))^2 dy$ when rotating around the y-axis or lines parallel to the y-axis, R(y) is the larger/outer radius and r(y) is the smaller/inner radius.

- Volume Using Cross Sections: This is hard to explain, but basically try to find a way to relate the area of the cross sections given to either $\int_a^b y_1 - y_2 dx$ or $\int_a^b x_1 - x_2 dy$. For example, if an object has square cross sections perpendicular to the x-axis then we would use the integral $\int_a^b (y_1 - y_2)^2 dx$ because the formula for the area of a square is s^2 and in this case $s=y_1-y_2$.

New Stuff:

- Arc Length: $\int_a^b \sqrt{1 + (f'(x))^2} dx$.

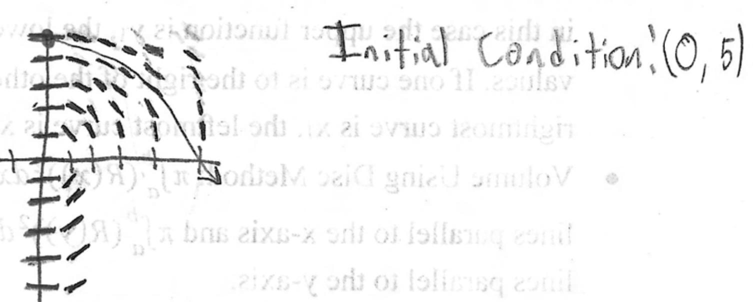
AP Exam:

When I took my AP exam we only had 1 MCQ question on arc length, 1 MCQ question on average value, and 3 or 4 questions on making 3D shapes out of functions. Typically, they'll give you a question where you'll have to washer method and they'll give you a few questions where you have to use some weird cross sections to make a 3D shape, like I had to use a right isosceles triangle as one of my cross sections.

6. Differential Equations

Old Stuff:

- Slope Fields: In short you take a differential equation like $\frac{dy}{dx} = \frac{-x}{y}$ and sketch a small little graph showing the values of the instantaneous rate of change at certain points. See example below.



- Separation of Variables: $\frac{dy}{dx}=x^2y$, $\frac{1}{y}dy=x^2dx$, $\ln(y)=\frac{x^3}{3}+C$, $y=Ce^{\frac{x^3}{3}}$. This is the general solution to the equation, however they may give you an initial condition, like $y(0)=1$, and ask you to find the particular solution. In this case just plug in the initial condition and solve for C then plug C into the general solution. For example, $1=Ce^0$, $C=1$, $y=e^{\frac{x^3}{3}}$.

New Stuff:

- Euler's Method: Basically find the equation for a tangent line using the differential equation and initial conditions given. Next, use this equation to approximate the next y-value and use this new point to make another tangent line and approximate the next value. You just keep repeating these steps using whatever the specified step size is until you get the final approximation. See example below.

n	x	y	dy/dx	tangent line	approximation
1 (initial)	1	1	2x/y=2	y=2x-1	y=2(1.5)-1=2
2	1.5	2	1.5	y=1.5x-0.25	y=1.5(2)-0.25= 2.75
3	2	2.75			

- Logistic Growth Model: $P(t)=\frac{L}{1+Ce^{-kt}}$ and $\frac{dP}{dt}=kP(1-\frac{P}{L})$. In the logistic growth model L is the carrying capacity, P is the population size, t is time, and k and C are constants. Additionally, $\frac{dP}{dt}$'s max value is L/2, this is nice to know for the exam.

AP Exam:

You'll probably end up getting at least one slope field question on the AP exam, and some FRQs require you to sketch a slope field then draw an estimate of the curve from it. Additionally, I got both Euler's method and logistic growth model questions on the MCQ portion of the exam, typically they only have you do a few steps for Euler's method.

7. Infinite Series and Sequences

- Geometric Series: $\sum_{n=1}^{\infty} A(r)^n$, If $|r|<1$ then the series converges to $\frac{A}{1-r}$ but if $|r|\geq1$ then the series diverges.
- nth Term Test: If $\lim_{n\to\infty} a_n$ equals anything except for 0 the series diverges, however if it equals 0 then the test is inconclusive.
- Integral Test: If a_n is positive, decreasing, and continuous over the interval then if $\int_k^{\infty} f(x)dx$ converges so does $\sum_{n=k}^{\infty} a_n$ and if $\int_k^{\infty} f(x)dx$ diverges so does $\sum_{n=k}^{\infty} a_n$. Note: $f(x)=a_n$ just change out the n's for x's.
- P-Series: $\sum_{n=1}^{\infty}(1/n)^p$, If p>1 then the series converges but if p≤1 then the series diverges. Also $\sum_{n=1}^{\infty}(\frac{1}{n})$ is called the harmonic series, which diverges.
- Limit Comparison test: If $\lim_{n\to\infty}\frac{a_n}{b_n}$=a finite positive number then both a_n and b_n either converge or diverge. Note: It doesn't matter what series goes on top.

- Direct Comparison Test: If $a_n \leq b_n$ and a_n diverges then so does b_n and if $a_n \geq b_n$ and a_n converges then so does b_n.
- Ratio Test: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$=L, if L<1 then $\sum_{n=k}^{\infty} a_n$ converges, if L>1 $\sum_{n=k}^{\infty} a_n$ diverges, and if L=1 then the test is inconclusive.
- Alternating Series Test: $\sum_{n=k}^{\infty}(-1)^n a_n$ converges if a_n is positive, decreasing, and $\lim_{n\to\infty} a_n$=0. It converges absolutely if a_n converges and converges conditionally if a_n diverges.
- Taylor Series: $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$. The Maclaurin series is just the Taylor series centered at 0.
- Lagrange Error Bound: $|R_n(x)| \leq \frac{M|x-a|^{n+1}}{(n+1)!}$, $|f^{(n+1)}(x)| \leq M$.
- Alternating Series Error Bound: $|R_n| \leq a_{n+1}$, if the remainder is positive it's an underestimate and if the remainder is negative it's an overestimate.
- Common Power Series: sin(x)=$\sum_{n=0}^{\infty}(-1)^n(\frac{x^{2n+1}}{(2n+1)!})$, cos(x)= $\sum_{n=0}^{\infty}(-1)^n(\frac{x^{2n}}{(2n)!})$, $e^x=\sum_{n=0}^{\infty}(\frac{x^n}{n!})$.
- Differentiating Power Series: Differentiate like normal and treat n like a constant. For example, $\frac{d}{dx}(\sum_{n=0}^{\infty}(-1)^n(\frac{x^{2n+1}}{(2n+1)!}))=\sum_{n=0}^{\infty}(-1)^n(\frac{(2n+1)x^{2n}}{(2n+1)!})$.
- Integrating Power Series: Integrate like normal and treat n like a constant. For example, $\int_0^1 \sum_{n=0}^{\infty}\frac{n+2}{2^n}x^{n+1}\,dx=\sum_{n=0}^{\infty}\left[\frac{x^{n+2}}{2^n}\right]_0^1=\sum_{n=0}^{\infty}\frac{1}{2^n}=\frac{1}{1-\frac{1}{2}}=2$.
- Geometric Series to Power Series: $\sum_{n=0}^{\infty} Ar^n$, A being the initial value and r being the common ratio. For example, f(x)=$\frac{1}{3+x^2}=\frac{1}{3}(\frac{1}{1+\frac{x^2}{3}})$, A=$\frac{1}{3}$ and r=$\frac{-x^2}{3}$, $\sum_{n=0}^{\infty}\frac{1}{3}(\frac{-x^2}{3})^n=\frac{1}{3}-\frac{x^2}{9}+\frac{x^4}{27}-\frac{x^6}{81}\ldots\frac{1}{3}(\frac{-x^2}{3})^n$.

AP Exam:

Infinite series and sequences are by far the largest part of the AP exam, so you should master this section. Additionally, you should learn how to differentiate/integrate power series and learn the power series for e^x, sin(x), and cos(x). I couldn't go completely in depth on all these topics since I'm trying to keep these guide brief, however Khan Academy has a great set of videos on this unit that I recommend watching.

8. Vector Valued Functions

- Differentiation: Simply differentiate the x and y components like normal. For example, p(t)=(t^2, ln(t)), v(t)=(2t, 1/t), a(t)=(2, $\frac{-1}{t^2}$).

- Integration: Like differentiation simply integrate the x and y components like normal. For example, a(t)=(2, $\frac{-1}{t^2}$), v(t)=(2t+C, $\frac{1}{t}$+C).
- Speed: The speed of a vector valued function is equivalent to the magnitude of its velocity, which can be calculated using the formula $\sqrt{(x'(t))^2 + (y'(t))^2}$.
- Distance: When calculating the distance of a vector valued function use the arc length formula $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 dt}$.

AP Exam:

Vector valued functions are about 3%-8% of the AP exam and you'll typically either get a vector valued function FRQ or a parametric equation FRQ on the AP exam, so you should definitely master this section. Mastering this section isn't hard, but remember that when it asks for the magnitude of anything you should use the formula $\overrightarrow{|f^n(t)|} = \sqrt{(x^n(t))^2 + (y^n(t))^2}$.

9. Parametric Equations

- Differentiation: When taking the first derivative of a parametric equation simply take the derivatives of the x and y components then plug them into the equation $\frac{y'}{x'}$. When taking higher order derivatives take the derivative of the previous order derivative then divide it by the derivative of the x component. For example, x(t)=t^4 and y(t)=e^t, x'(t)=4t^3 and y'(t)=e^t, $\frac{y'}{x'}=\frac{e^t}{4t^3}$, $\frac{y''}{x''}=(\frac{4t^3e^t-12t^2e^t}{16t^6})/4t^3$.
- Distance: When calculating the distance of a parametric equation use the arc length formula $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 dt}$.

AP Exam:

This section is very brief and you really only need to know the two concepts I explained above. I only had 2 or 3 MCQ questions on this topic on my AP exam, however some years there is an FRQ on parametric equations so they're worth knowing.

10. Polar Coordinates

- x(θ)=rcos(θ) and y(θ)=rsin(θ).
- Differentiation: When differentiating polar functions you follow the exact same processes as when differentiating parametric equations, so $\frac{y'}{x'}$.
- Area of a Polar Curve: $\frac{1}{2}\int_a^b r^2 d\theta$, the bounds are angles going counterclockwise.

- Area Between Polar Curves: This is difficult to explain but usually you'll have to take the area of one polar curve and subtract it from another or you'll have to split regions into multiple integrals and add them together to get a total area.

AP Exam:

Polar coordinates are a small part of the AP exam, I only got a few MCQ questions on them, however it's still important to learn them because you may get an FRQ on them. This section is pretty easy, however you have to understand how to split certain regions into multiple integrals so that you can find the area. Additionally, you'll need to understand how the integral bounds using angles work, REMEMBER YOU CAN ONLY MOVE COUNTERCLOCKWISE FROM BOUNDS A TO B.

Reflection Essay

AP Calculus BC can sound intimidating, especially if you struggled with AP Calculus AB, but it's much easier than you think it is. You really only have to learn two big new units which contain infinite series and sequences, polar coordinates, vector valued functions, and parametric equations. Additionally, you have a whole year of double block periods to do this, giving you plenty of time to master the old AB calc concepts that you may have struggled with and the new BC calc concepts. My advice would be to spend the first 2 quarters reviewing all 8 of the AB calc units and learning some of the new BC calc concepts as you go along, specifically integration by parts, integration using partial fraction decomposition, improper integrals, arc-trig integration, arc length, Euler's method, and logistic growth models. Then spend the 3rd quarter learning about the new BC calc units in this order: vector valued functions, parametric equations, polar functions, and infinite series and sequences. The hardest part of these units is memorizing all of the tests associated with infinite series and sequences, but I listed them all above in concept 7 so hopefully that helps. If you follow this pace then you'll have the entire 4th quarter to review for the AP exam, so take plenty of practice tests to get used to the types of questions that you'll see on the AP exam. If you happen to struggle with a concept at any point, I would recommend checking out Khan Academy because they have extremely helpful videos covering every aspect of BC calc. Furthermore, when learning new concepts, I would recommend completing the Khan Academy unit for it first then using the problems from the textbook to practice because the Khan Academy practice makes sure that you have an understanding of the concepts and the textbook challenges you to use your knowledge to solve some pretty weird questions that require you to think outside of the box.