CALCULUS : TEST 1 (LIMITS) STUDY GUIDE

Show work for full credit. No Calculators

Instructions: Find the limits:

① $\lim_{x \to 5} (-x^2 + 5x + 4)$

② $\lim_{x \to 7} \dfrac{\sqrt{6x+5}}{x-7}$

③ $\lim_{x \to 0^-} f(x)$, where $f(x) = \begin{cases} e^x ; & x < 1 \\ x+3 ; & x \geq 1 \end{cases}$

④ $\lim_{x \to 2} \cos(\pi x)$

⑤ $\lim_{x \to 0} \dfrac{\sqrt{x+9} - 3}{x}$

⑥ $\lim_{x \to 5} \dfrac{x-5}{x^2 - 25}$

⑦ $\lim_{x \to 1} h(x)$, where $h(x) = \begin{cases} 3x - 1 ; & x \leq 1 \\ 2x^2 ; & x > 1 \end{cases}$

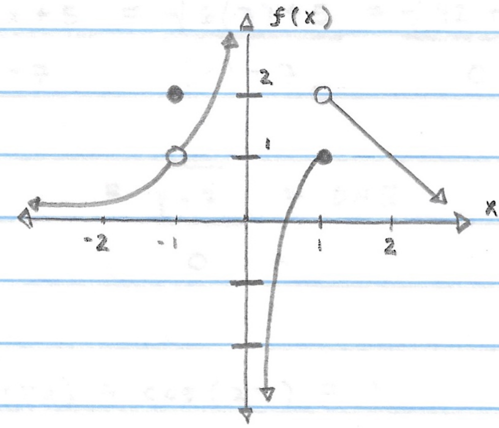
⑧ True or False: If there exists a V.A. @ $x = c$, then the limit of $f(x)$ as x approaches c does not exist.

⑨ Find a c such that $f(x)$ is continuous on the entire real number line. $f(x) = \begin{cases} 3x^2 ; & x \leq 4 \\ cx ; & x > 4 \end{cases}$

M.C. ⑩ If $a \neq 0$, then $\lim_{x \to a} \dfrac{x - a}{x^2 - a^2}$ is:

a) 0 b) DNE c) $\dfrac{1}{x+a}$ d) $\dfrac{1}{2a}$

Use the graph at the right to answer 11 – 14 :



(11) $\lim\limits_{x \to -1} f(x) =$

(12) $\lim\limits_{x \to 1^+} f(x) =$

(13) $\lim\limits_{x \to 0^-} f(x) =$ (14) $\lim\limits_{x \to 1^-} f(x) =$

(15) Find the x-values (if any) at which $f(x) = \dfrac{3(x+2)}{x^2 - 4}$ is discontinuous.

(16) Determine all the vertical asymptotes of $g(x) = \dfrac{x+3}{x^2 - 9}$.

(17) AP Test ? (Multiple Choice), No hints allowed

(18) Is $f(x) = 3x - 1$ continuous at $x = 2$? Use the def'n of continuity to justify your answer.

(19) Let $f(x) = x^2 + 2x + 8$. Explain, using IVT, why there must exist a value c for $c \in [-1, 4]$ such that $f(c) = 11$. Find c.

(20) Explain, using IVT, why the function $f(x) = x^3 - 2x^2 + 2$ must have a zero in the interval $[-3, 1]$.

CALCULUS TEST 1 S.G. SOLUTIONS

① $$\lim_{x \to 5} (-x^2 + 5x + 4)$$
$$\Rightarrow -(5)^2 + 5(5) + 4$$
$$-25 + 25 + 4$$
$$4$$

② $$\lim_{x \to 7} \frac{\sqrt{6x+5}}{x-7} = \frac{\sqrt{6(7)+5}}{(7)-7} = \frac{\sqrt{42+5}}{0}$$
$$= \frac{\sqrt{47}}{0} = \text{DNE}$$

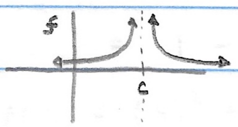
③ $$\lim_{x \to 0^-} f(x) = e^0 = 1$$

④ $$\lim_{x \to 2} \cos(\pi x) = \cos(2\pi) = 1$$

⑤ $$\lim_{x \to 0} \frac{(\sqrt{x+9} - 3)}{x} \cdot \frac{(\sqrt{x+9} + 3)}{(\sqrt{x+9} + 3)} \quad \lim_{x \to 0} \frac{\cancel{(x+9)} - \cancel{9}}{\cancel{x}(\sqrt{x+9} + 3)} \quad \lim_{x \to 0} \frac{1}{\sqrt{x+9} + 3} = \frac{1}{6}$$

⑥ $$\lim_{x \to 5} \frac{x-5}{x^2-25} \quad \frac{0}{0} \quad \lim_{x \to 5} \frac{\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} \quad \lim_{x \to 5} \frac{1}{(x+5)} \quad \frac{1}{5+5} \quad \frac{1}{10}$$

⑦ $$\lim_{x \to 1^-} h(x) = 3(1) - 1 = 2 \quad \lim_{x \to 1^+} h(x) = 2(1)^2 = 2 \quad \lim_{x \to 1} h(x) = 2$$

⑧ 

$$\lim_{x \to c} f(x) = \text{DNE}$$
TRUE

⑨ $$3x^2 = cx \text{ for } x = 4 \quad 3(4)^2 = c(4) \quad 3(16) = 4c \quad c = \frac{3(16)}{4} \quad c = 3(4) \quad c = 12$$

⑩ $$\lim_{x \to a} \frac{x-a}{x^2-a^2} = \frac{a-a}{a^2-a^2} = \frac{0}{0} \quad \lim_{x \to a} \frac{1\cancel{(x-a)}}{(x+a)\cancel{(x-a)}} \quad \frac{1}{a+a} = \frac{1}{2a} \quad d$$

(11) $\lim_{x \to -1} f(x) = 1$

(12) $\lim_{x \to 1^+} f(x) = 2$

(13) $\lim_{x \to 0^-} f(x) = \text{DNE}$

(14) $\lim_{x \to 1^-} f(x) = 1$

(15) f is discontinuous when denominator equals zero, so...

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

(16) $g(x) = \dfrac{x+3}{x^2-9} = \dfrac{1\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$

$g(x) = \dfrac{1}{x-3}$

V.A. @ $x = 3$

(17) Good Luck ☺

(18) Since $f(2) = 5 = \lim_{x \to 2} f(x)$, f is continuous @ $x = 2$ by def'n of continuity.

(19) $f(-1) = (-1)^2 + 2(-1) + 8$

$= 1 - 2 + 8 = 7$

$f(4) = (4)^2 + 2(4) + 8$

$= 16 + 8 + 8 = 32$

Since f is continuous on $[-1, 4]$, $f(-1) = 7 \neq 32 = f(4)$ & $11 \in (7, 32)$, then there must exist at least one $c \in (-1, 4)$ s.t. $f(c) = 11$ by IVT. In particular, $c = 1$.

$11 = x^2 + 2x + 8$

$0 = x^2 + 2x - 3$

-3, 3, -1, 2

$0 = (x+3)(x-1)$

$x = \cancel{-3}, 1$

(20) $f(-3) = -27 - 18 + 2$

$= -45 + 2 = -43$

$f(1) = 1 - 2 + 2 = 1$

Since f is continuous on $[-3, 1]$, $f(-3) = -43 \neq 1 = f(1)$, and $0 \in (-43, 1)$, then there must exist at least one $c \in (-3, 1)$ s.t. $f(c) = 0$ by IVT.