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pd 4
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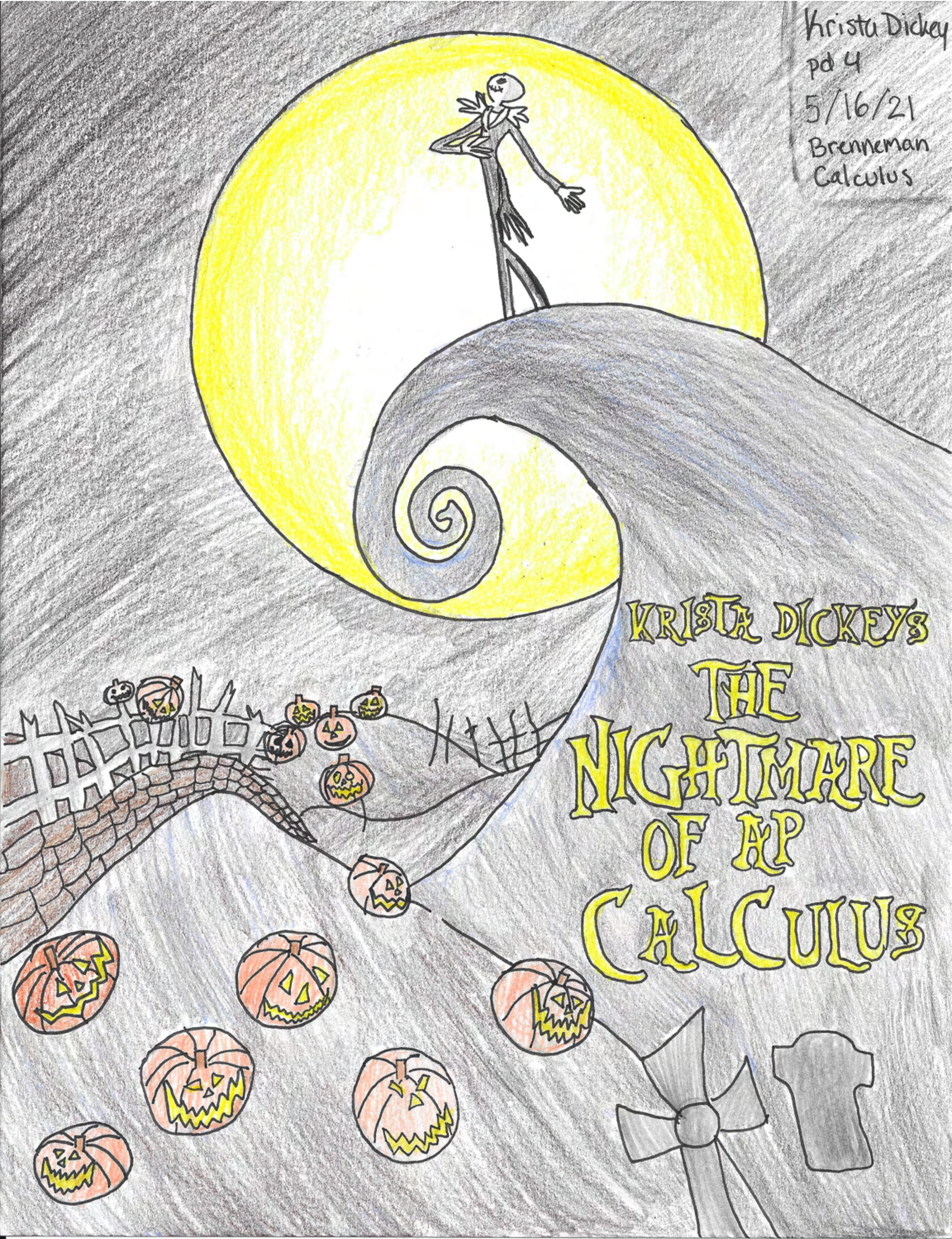
Brenneman
Calculus



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① LIMITS

The limit of a function is the value of the approaching y-variable as x approaches a fixed value

Basic Form: $\lim_{x \to c} f(x) = ?$

↳ This means "The limit of f(x) as x approaches c"

STEPS to Solving Limits

1. **Direct Substitution** (Plug the value into the equation and solve)

Example 1: $\lim_{x \to -2} \frac{3x+1}{2-x} \Big| \frac{3(-2)+1}{2-(-2)} = \frac{-4}{4} \Big| \lim_{x \to -2} \frac{3x+1}{2-x} = -1$

Example 2: $\lim_{x \to -3} \frac{x^2+x-6}{x+3} \Big| \frac{(-3)^2+(-3)-6}{-3+3} = \frac{0}{0}$ ← Indeterminate Form

↳ If you get $\frac{0}{0}$ you must manipulate the equation and plug the value back in

2. **Factoring**

Example 1: $\lim_{x \to -3} \frac{x^2+x-6}{x+3} \Big| \frac{(-3)^2+(-3)-6}{(-3)+3} \Big| \frac{0}{0} \Big|$ 3 × −2, 6, 1

$\lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)} \Big| \lim_{x \to -3} x-2 = -5$

Example 2: $\lim_{x \to 2} \frac{x^2+4x-12}{x^2-2x} \Big| \frac{(2)^2+4(2)-12}{(2)^2-2(2)} = \frac{0}{0} \Big| \lim_{x \to 2} \frac{(x-2)(x+6)}{x(x-2)} = 4$

3. **Rationalizing** (Multiply top & bottom by the reciprocal)

Example: $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} \Big| \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} \Big| \frac{(x-9)}{(x-9)(\sqrt{x}+3)} \Big| \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

② DERIVATIVES

A derivative is the slope of the tangent line

The Basic Rules:

1. **Constant:** $\frac{d}{dx}(c) = 0$

↳ Ex: $\frac{d}{dx}(5) = 0$

2. **Power:** $\frac{d}{dx}(x^n) = nx^{n-1}$

↳ Ex: $x^4 \mid 4x^3$

3. **Chain:** $\frac{d}{dx}(f(g(x)))$

↳ Ex: $(3x+1)^5 \mid 5(3x+1)^4 \cdot (3)$

$15(3x+1)^4$

4. **Product:** $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$

↳ Ex: $(x^2)(x^3+4) \mid (x^2)(3x^2) + (x^3+4)(2x)$

$3x^4+2x^4+8x \mid 5x^4+8x$

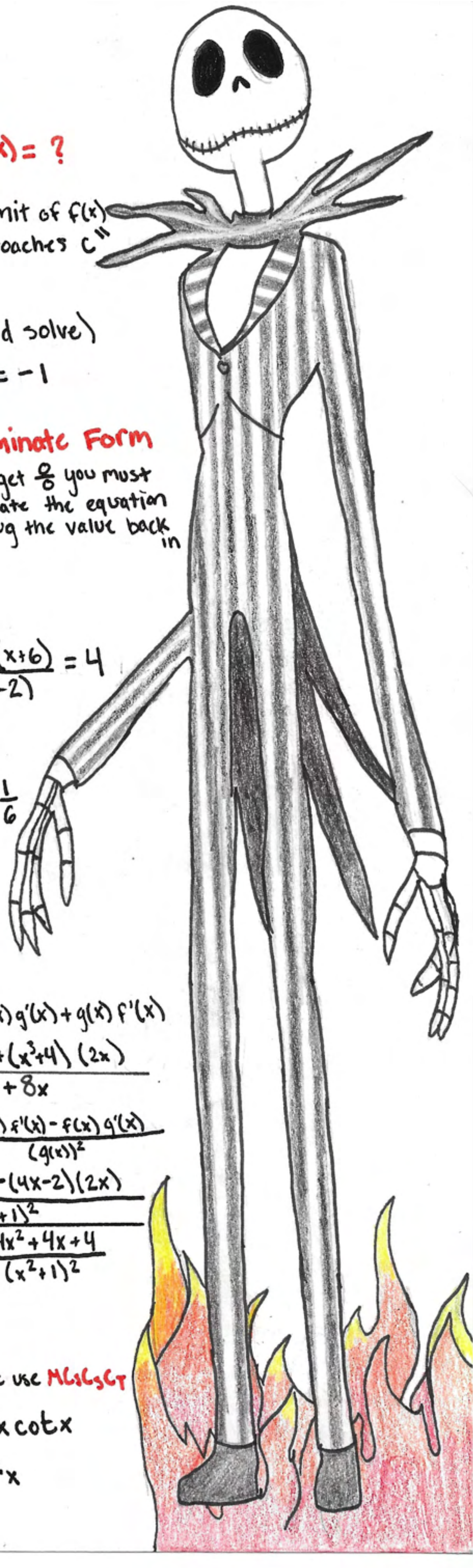
5. **Quotient:** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

↳ Ex: $\left(\frac{4x-2}{x^2+1}\right) \Big| \frac{(x^2+1)(4)-(4x-2)(2x)}{(x^2+1)^2}$

$\frac{(4x^2+4)-(8x^2-4x)}{(x^2+1)^2} \Big| \frac{-4x^2+4x+4}{(x^2+1)^2}$

Trig

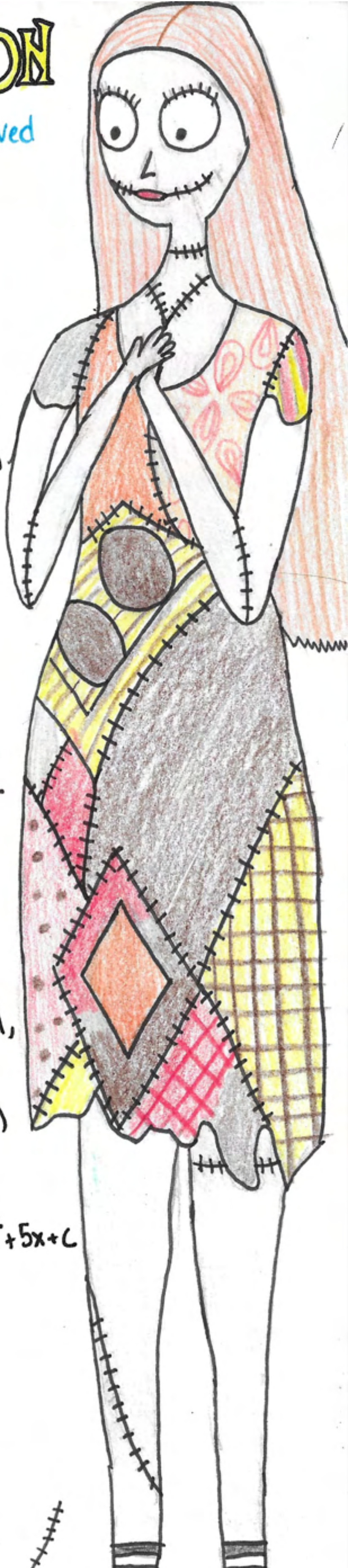
	If there is tan or sec use **PSST**	If there is csc or cot use **MCsCsCT**
$\frac{d}{dx}\sin x = \cos x$	$\tan x = \sec^2 x$	$\csc x = -\csc x \cot x$
$\frac{d}{dx}\cos x = -\sin x$	$\sec x = \tan x \sec x$	$\cot x = -\csc^2 x$



③ IMPLICIT DIFFERENTIATION

This is the process to find $\frac{dy}{dx}$ when y is not easily solved

Explicit	Implicit
$y = 6x^2 + 12x$	$x^4 - 3y^2 + 2y = 55$



STEPS for Implicit Differentiation

1. Take the derivative of both sides with respect to x
2. Collect terms with dy/dx on one side of the equation
3. Factor out dy/dx
4. Solve for dy/dx

Example 1: $x^2+y^2=16$ | $2x + 2y \cdot \frac{dy}{dx} = 0$ | $2y \cdot \frac{dy}{dx} = -2x$ | $\frac{dy}{dx}(2y) = -2x$ | $\frac{dy}{dx} = \frac{-2x}{2y}$ | $\frac{dy}{dx} = \frac{-x}{y}$

Example 2: $x^3 + 3y^2 + y^3 = 8$ | $3x^2 + 6y \cdot \frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ | $6y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2$ | $\frac{dy}{dx}(6y+3y^2) = -3x^2$ | $\frac{dy}{dx} = \frac{-3x^2}{6y+3y^2}$

④ INTEGRATION

Integration means the area under the curve

When solving an integral, always put "+c" at the end (c = constant)

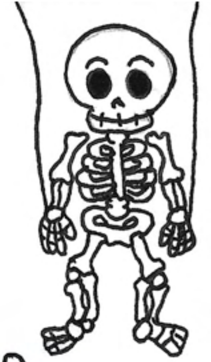
Integration Rules

$\int 0\, dx = c$
$\int k\, dx = k + c$
$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$
$\int 1/x\, dx = \ln\lvert x\rvert + c$
$\int \sin x\, dx = -\cos x + c$
$\int \cos x\, dx = \sin x + c$
$\int \sec^2 x\, dx = \tan x + c$
$\int \csc x \cot x\, dx = -\csc x + c$
$\int \sec x \tan x\, dx = \sec x + c$
$\int \csc^2 x\, dx = \cot x + c$

Examples:

1. $\int(x^2-4x+5)$ | $\frac{x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + 5x + c$ | $\frac{x^3}{3} - 2x^2 + 5x + c$
2. $\int(10x^4+6x^2-2)$ | $\frac{10x^5}{5} + \frac{6x^3}{3} - 2x + C$ | $2x^5 + 2x^3 - 2x + c$
3. $\int(x^3 + \cos x + 1)$ | $\frac{x^4}{4} + \sin x + x + c$
4. $\int(\sec^2 x + \csc^2 x)$ | $\tan x - \cot x + c$
5. $\int(\frac{1}{x} + e^x)$ | $\ln\lvert x\rvert + e^x + c$

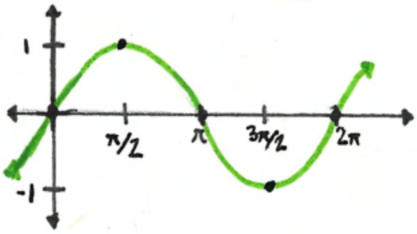
⑤ F' GRAPHS



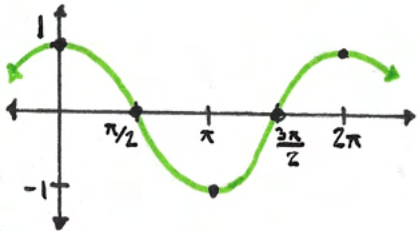
f' graphs are used by analyzing the derivative of a function to find the qualities of the original

The easiest way to see an F and F' graph is using the base function sin x.

$f(x) = \sin x$



$f'(x) = \cos x$



Qualities of f' that leads to f

1. f(x) is increasing when $f' > 0$
   f(x) is decreasing when $f' < 0$
2. f(x) has a rel. max when f' changes from pos to neg
   f(x) has a rel. min when f' changes from neg to pos
3. f(x) is concave up when f' is increasing
   f(x) is concave down when f' is decreasing
4. f(x) has an inflection point when f' changes from increasing to decreasing or vice versa.

{ There is always an f' graph frq on the AP Test so if you know these basic qualities you can get a couple points. }

⑥ CHAIN RULE

Chain Rule is the derivative of the outside times the derivative of the inside → $f(g(x)) = f'(g(x)) \cdot g'(x)$

STEPS to Chain Rule

1. Exponent becomes a coefficient
2. Subtract 1 from the exponent
3. Differentiate the content of the bracket and multiply it by the above steps

Example 1: $(2x^3+2x)^5 \mid 5(2x^3+2x)^4 \cdot (6x^2+2)$

subtract one

Exponent becomes coefficient

Derivative of the inside

Example 2: $(5x^3+3x)^3 \mid 3(5x^3+3x)^2 \cdot (15x^2+3)$

The same method applies to Trig

Ex: $\cos(2x) \mid -\sin(2x) \cdot (2)$

Derivative of the outside

Derivative of the inside

⑦ RIEMANN SUMS

Riemann Sums estimate the area under the curve

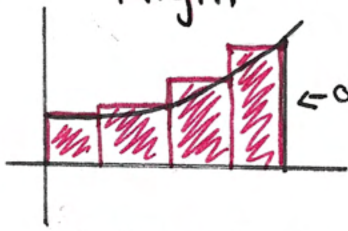
4 types of Riemann Sums:

Left

← Underestimate

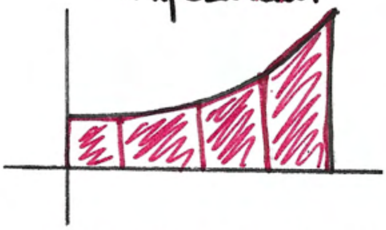
Midpoint

Right



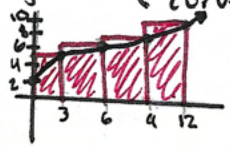
← overestimate

Trapezoidal



To solve a Riemann Sum you want to add the are of each rectangle together (Area under curve: Area of R1 + A of R2 + A of R3 + A of R4)

Example: (Right Riemann sum)



$A = (3)(5) + (3)(7) + (3)(8) + (3)(10)$

$15 + 21 + 24 + 30$

$\underline{90}$

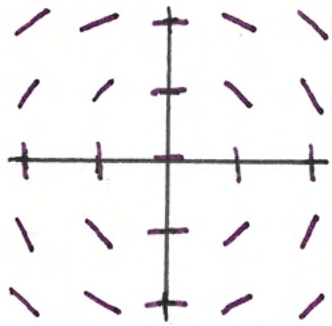
⑧ SLOPE FIELDS

Slope Fields show the shape a function could take at various values

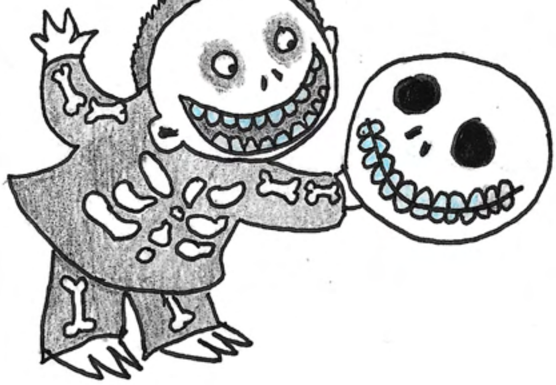
To create a slope field, select a value of (x, y) and plug it into the function to find the slope at that point. Graph that slope at the specified point.

Example: $\frac{dy}{dx} = \frac{-x}{y}$

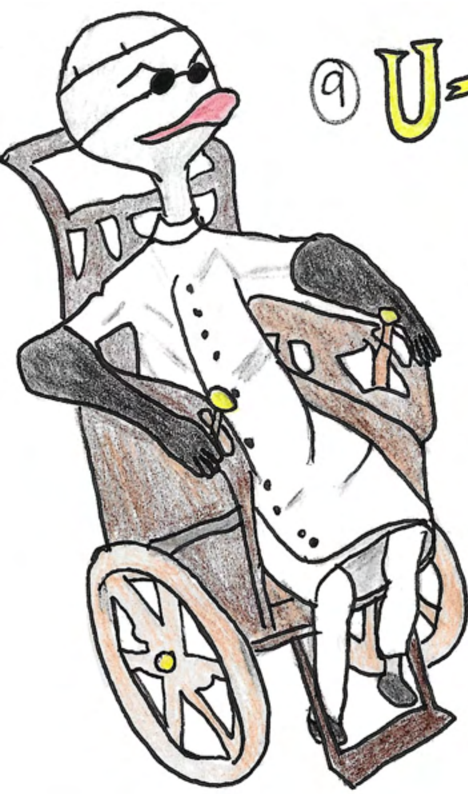
(x,y)	-2	-1	0	1	2
-2	-1	-2	Ø	2	1
-1	-½	-1	Ø	1	½
0	0	0	Ø	0	0
1	½	1	Ø	-1	-½
2	1	2	Ø	-2	1



Take your time finding the values, mistakes are easily made!



↳ Solve by plugging in: $\frac{-(-2)}{-2} = -1$



⑨ U-SUBSTITUTION

U-Substitution is a quicker way to solve an integral

Example: $\int (3x^2+1)(6x)\,dx$

$U = 3x^2+1$
$dU = 6x\,dx$ ← The U-value is what is in the parenthesis. The dU-value is the derivative of U

$\int U\,dU \;\Big|\; \frac{U^2}{2}$ ← Take the integral using U

$\frac{(3x^2+1)^2}{2}$ ← Plug U into the integral

Sometimes an equation will have a du that does not match what is left. To fix this you must alter the function and cancel the change with a reciprocal

Example: $\int x(x^2+2)\,dx \;\Big|\; \begin{matrix} u = x^2+2 \\ du = 2x\,dx \end{matrix} \;\Big|\; \frac{1}{2}\int 2x(x^2+2)\,dx$

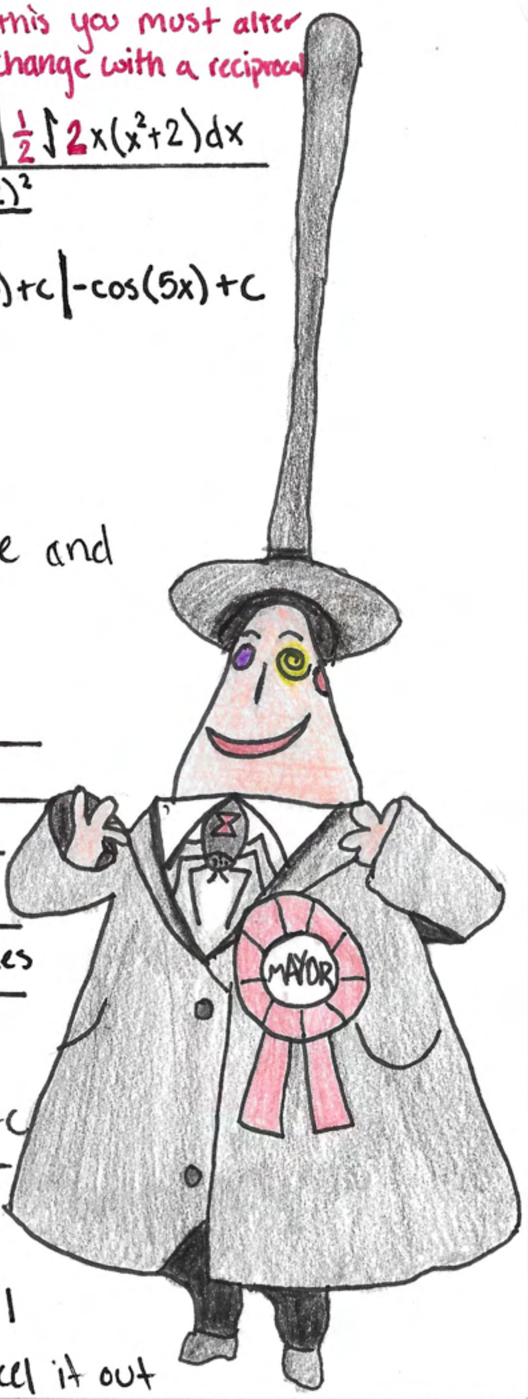
$\frac{1}{2}\int u\,du \;\Big|\; \frac{1}{2}\cdot\frac{u^2}{2} \;\Big|\; \frac{1}{2}\cdot\frac{(x^2+2)^2}{2}$

Trig: $\int 5\cdot\sin(5x)\,dx \;\Big|\; \begin{matrix} u = 5x \\ du = 5\,dx \end{matrix} \;\Big|\; \int \sin(u)\,du \;\Big|\; -\cos(u)+c \;\Big|\; -\cos(5x)+c$

⑩ DIFFY Q's

Diffy Q stands for differential equation

To solve a diffy Q you must get all x's on one side and all y's on the other, then solve for y



Example 1:

$y\cdot\frac{dy}{dx}-(x^2+5)=0$	Add (x^2+5) to both sides
$y\cdot\frac{dy}{dx}=(x^2+5)$	Multiply both sides by dx
$\int y\cdot dy=\int(x^2+5)\,dx$	Integrate both sides
$\frac{y^2}{2}=\frac{x^3}{3}+5x+C$	Multiply by 2
$y^2=\frac{2x^3}{3}+10x+C$	Take square root of both sides
$y=\sqrt{\frac{2x^3}{3}+10x+C}$	

Example 2: $\frac{dy}{dx}=3y \;\Big|\; dy=3y\,dx \;\Big|\; \int\frac{1}{y}\,dy=\int 3\,dx \;\Big|\; \ln|y|=3x+c$

$e^{\ln|y|}=e^{3x+c} \;\Big|\; |y|=e^{3x}+e^{c} \;\Big|\; y=ce^{3x}$

↳ If there is $\frac{1}{y}$, $\frac{1}{x}$, or $\frac{1}{u}$ you must use $\ln|\;|$ to integrate, then raise it to e to cancel it out

REFLECTION ESSAY

AP Calculus. What an interesting class to say the least. It seems very easy in the beginning. Progressively it gets harder so be sure to ask for help directly instead of mentioning to the class you are confused. To be successful in this class make sure to pay attention to the notes and do the homework. In this class, I love to complain. For me it makes the work seem easier when I do. Find something that works for you to make the class easier. While this has not been my favorite class, I hope you enjoy it and that you do well. (Fight for that A).

From the student who ended up with an A,

-Krista