5/10/19 - 6th / 7th - AP Calculus AB - Brenneman

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AP CALCULUS AB FINAL PROJECT RUBRIC

Instructions: You are to develop a Calculus Study Guide that will help future calculus students review key concepts for their AP Test. Your Study Guide needs to have a Top 10 List of what **you** think are the most important concepts to know going in to the test. Your study guide can be handwritten or typed, but it must be written on **no more than five** 8.5 x 11 inch computer/resume papers. (The rubric does not count as one of your five pages). You may use the front & back sides of each paper. I would prefer you to handwrite your project, but if you have sloppy handwriting, then you should type it.

On the last page of your Study Guide (after Key Concept 10), I want you to write a reflection essay (comprising of at least one paragraph), of what you think future Calculus students should do to not only be successful for the AP Calculus Test, but to also be successful in class. You can talk about the importance of attendance, completing homework, not falling behind, maintaining a positive attitude, getting extra help after school, etc. Don't be lazy here. Extra Credit will be awarded to students who go above and beyond the following requirements. **Please print out this Project Rubric, put your name on the top right corner of this page, and place all your work in a page protector. Have this rubric as the first page. Please DO NOT staple anything.**

Final Project Point Breakdown:

10	0 - 10	Name, Date, Period, Subject, and Teacher's Last Name as the Header.
10	0 - 10	10 Key Calculus Concepts which you think are a must know for the AP Test. (Must be numbered).
10	0 - 10	Elaboration of each key concept (Could be an example, a picture, a detailed paragraph explaining each concept, etc. or all of the above).
10	0 - 10	Elaboration of each key concept makes sense (Examples have correct solutions, explanations are accurate, pictures are relevant, spelling is correct, etc.).
10	0 - 10	Has original ideas. All work isn't just copied out of the notebook/homework.
10	0 - 10	Project is on no more than five 8.5 x 11 inch computer/resume papers. **(No loose leaf paper)**.
10	0 - 10	Neat, colorful, well designed, utilizes space provided. Nothing glued or taped.

Reflection Essay Breakdown:

10	0 - 10	Reflection Essay is located after Key Concept 10.
20	0 - 20	Reflection Essay is well written, spelling/grammar is correct, ideas make sense, reflections reference personal experiences and has a motivational tone to the reader.
105	/ 100	[Final Score]

Project Due Date: On Website

LIMITS

Objective: Calculate y-variable value as the function approaches a fixed 'x' value

①

Find $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} - 2$ Try D.S 1^{st} Always $\rightarrow \frac{0}{0} - 2 = \emptyset$

$\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} - 2 \xrightarrow{\text{D.S.}} 4 - 2 = 2$

Note: The limit is not dependant on Continuity

If $\frac{0}{0}$ occurs L'Hopitals rule applies

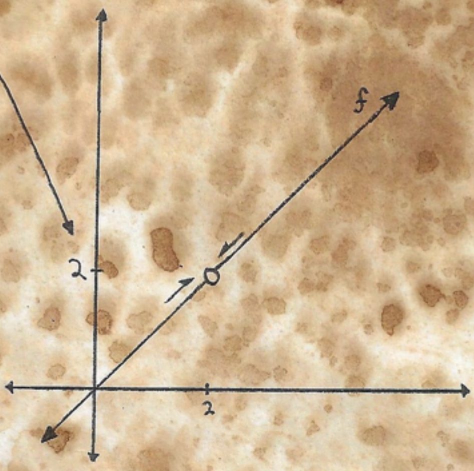


Figure 1.1

L'HA: $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} - 2$

- Take Derivative of Top and Bottom of fraction independently
  - Do NOT use Quotient Rule

$\frac{\frac{d}{dx}[x^2 - 4]}{\frac{d}{dx}[x - 2]} - 2 \rightarrow \frac{2x}{1} - 2 \xrightarrow{\text{D.S.}} 2$

Toolbox:

- Direct Substitution
- Canceling Common Factors
- Rationalize Denominator
- L'Hopitals Rule ! Advanced Skill!
  - Requires Differentiation

DIFFERENTATION

②

Math Juice | 2+2=5

Limit Def'n of Derivative:

$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

This is used to Find the Instantaneous rate of change of a function

Tools:

- Constant Rule: $\frac{d}{dx}[c] = 0$, Always.
- Power Rule: $\frac{d}{dx}[x^n] = n \cdot x^{n-1}$
  Your Best Friend
- Scalar Multiple: $\frac{d}{dx}[c \cdot f(x)] \rightarrow c \cdot \frac{d}{dx}[f(x)] \rightarrow c \cdot f'(x)$
  Denotes Derivative
- Chain Rule: $\frac{d}{dx}[f(g(x))] \rightarrow f'(g(x)) \cdot g'(x)$

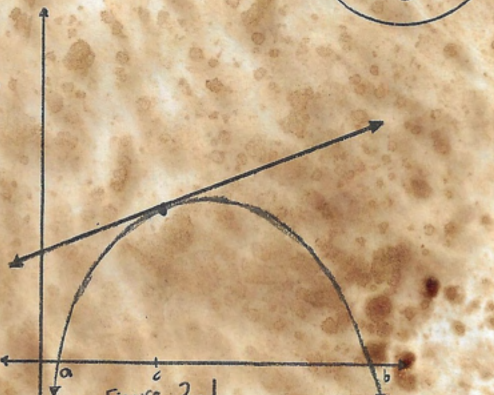


Figure 2.1

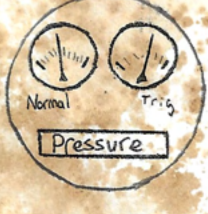
This is a tangent line hitting only one point of the graph, showing the exact slope at precisely $x = c$

- Trig: $\sin x \rightarrow \cos x$; $\cos x \rightarrow -\sin x$
  - No method Just Memorize

Positive Secant Secant Tangent "Derivative of one is the other Two" Negative Cosecant Cosecant Cotangent

- Product Rule: First · Second' + Second · First'
- Quotient Rule: $\frac{(\text{Low}) \cdot (\text{D-High}) - (\text{High})(\text{D-Low})}{(\text{Low})^2}$
  Cowboy Voice

INTEGRATION



- Work Order: Use Anti-Derivatives to Calculate the Exact Area under the Curve on [a, b]

Indefinite

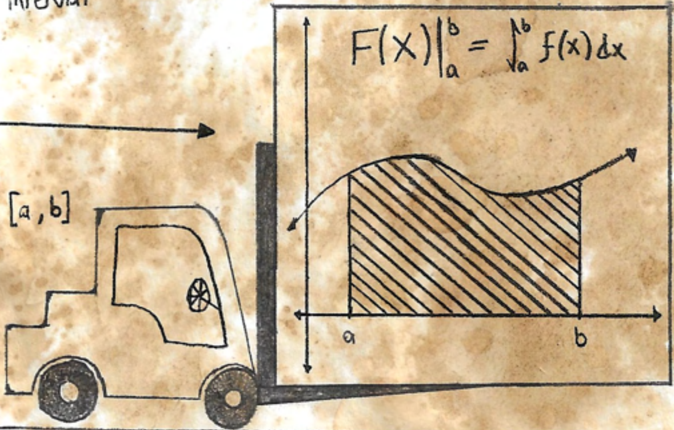
- Just a way for going backwards and acheiving the Anti-Derivative

Note: Needs a "+ C"

Definite

- Used to calculate area under curve over a specific interval using FTOC

Doing This →



Tools:

- FTOC: $\int_a^b f(x) = F(b) - F(a)$ = Area under Curve on [a, b]
- Reverse Power Rule $\int x^n\,dx \rightarrow \frac{x^{n+1}}{n+1} + C$

Note: Doesn't work on Trig and if RPR doesn't work Try U-Sub.

- U - Substitution: $\int x(x^2+2)\,dx \Rightarrow \left(\frac{1}{2}\right)\int u\,du$

$u = x^2+2$
$du = 2x\,dx$

Note: "du" is always derivative of "u" so sometimes you need to multiply #'s such as the "1/2" and "2" in the example

Trig: "PSST" and "$MC_sC_sC_T$" (Mist) also work with AD's
But this time every 2 is the other 1.

Ex: $\int \sec^2 x\,dx = \tan x + C$ } where "c" is some constant

SKETCHING

$f'(x) = \rightarrow \frac{x^2-5}{x^2}$

use quotient Rule, then use f' to find CN's ($f'=0, \emptyset$)

CN's: $x = 0, \pm\sqrt{5}$

Also make test interval to find when decreasing / increasing

+ − | − +

$-\sqrt{5}$ | 0 | $\sqrt{5}$

Rel. Max @ $x = -\sqrt{5}$ | Rel. Min @ $x = \sqrt{5}$

$f'' = \frac{10}{x^3}$

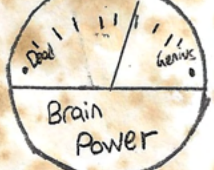
$f'' \neq 0$ PIP
$f'' = \emptyset$ @ $x = 0$

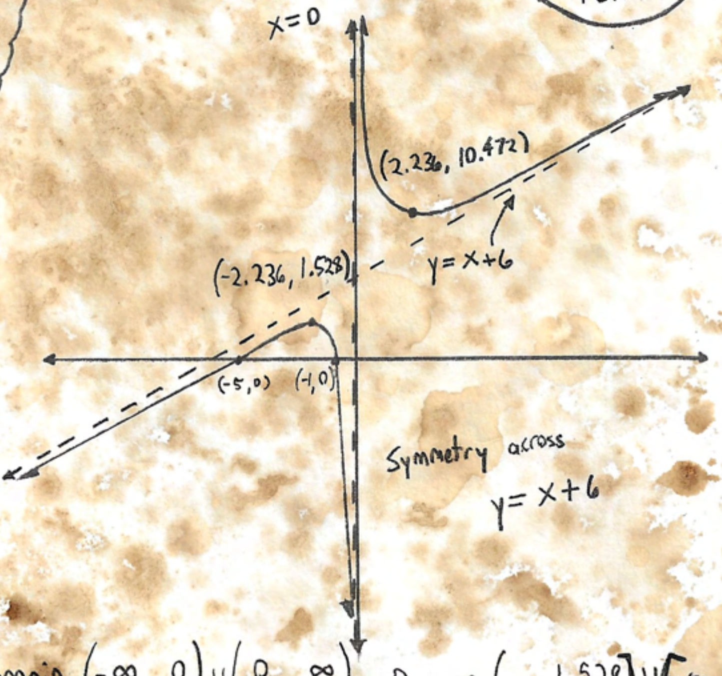
Similarly Test intervals To test for concavity

− | +

$$\begin{array}{r} x + 6 + \frac{5}{x} \\ x \enclose{longdiv}{x^2+6x+5} \\ -x^2 \downarrow \\ \hline 0+6x \end{array}$$

Graph $f(x) = \frac{x^2+6x+5}{x}$





Symmetry across $y = x+6$

Asymptotes:

V.A.: $x = 0$

H.A.: None, Since numerator degree > denominator

But! Slant Asymptote!

S.A.: $y = x+6$

Domain $(-\infty, 0) \cup (0, \infty)$ Range $(-\infty, 1.528] \cup [10.472, \infty)$

PARTICLE IN MOTION

Speeding ↑/↓: Knowing velocity and acceleration is key

If velocity and acceleration have same sign then speeding ↑

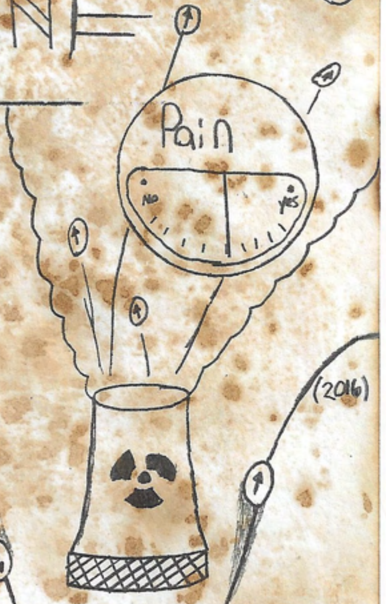
If $v(t)$ and $a(t)$ have different sign then slowing ↓

Distance: $\int_a^b |v(t)|\,dt$ = Total Distance Traveled

Displacement: $\int_a^b v(t)\,dt$ = Total Net Change

Similarly to F' Graphs use FTOC with initial conditions to go backwards ($v(t) \rightarrow x(t)$) to find particle's positions

Change of Direction: when $v(t)$ passes through the t-axis



(2016)

$x(t) \xrightarrow{\frac{d}{dt}} v(t) \xrightarrow{\frac{d}{dt}} a(t) \xrightarrow{\frac{d}{dt}} j(t)$

(also $s(t)$) Position — Velocity — Acceleration — Jerk (won't do in Calc)

Velocity: Speed with direction (if "−" then moving left; if "+" then moving right)

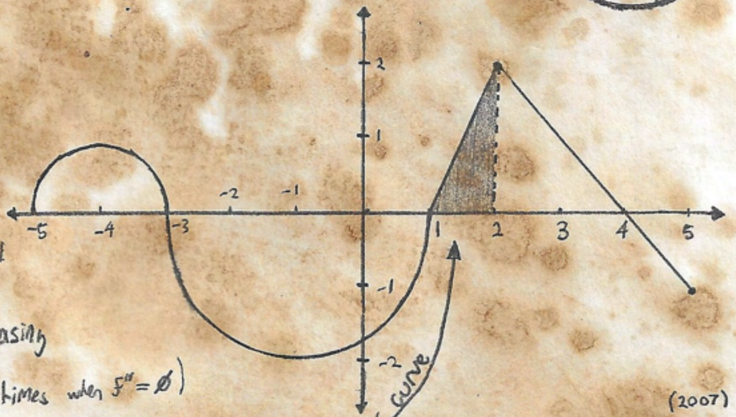
Acceleration: Rate of change of velocity

Think of a CAR  with Acceleration and Deceleration

F' GRAPHS

- f is defined on $[-5, 5]$ $f(1) = 3$
  The graph f' is constructed with Two semi-circles and two line segments

- **Work Training:**
- For the graph of f', all negative y-values (under x-axis) is when f is decreasing at that x-value. Alternatively all positive y-values of f' are increasing at that x-value.
  - Example: f is decreasing on $(-3, 1) \cup (4, 5)$
- **Extrema:** Extrema can be found when f' passes through the x-Axis
  $+ \rightarrow -$ means Rel. Max $\quad - \rightarrow +$ means Rel Min.
- **Inflection points:** Points of inflections can be found when f' has relative extrema when f' changes from increasing to decreasing
  - Think of when f' changes from increasing to decreasing or vice versa
  - Or think when f' has a horizontal tangent (or sometimes when $f'' = \emptyset$)
- **Concavity:** $f(x)$'s concavity can be determined when f' is increasing or decreasing. If f' is increasing, f is concave up. If f' is decreasing, f is concave down



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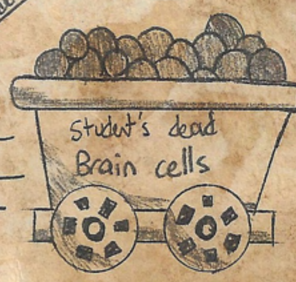


- **Finding Points of $f(x)$:** You can use an initial condition and FTOC to find coordinates of f. What's $f(2) = ?$ we know $f(1) = 3$ so using FTOC we can solve it

$$\int_1^2 f'(x) = f(2) - f(1) \Rightarrow \int_1^2 f'(x) = f(2) - 3 \rightarrow f(2) = 3 + \left(\frac{1 \cdot 2}{2}\right) = 4$$

DIFFY Q's

w/ IMPLICIT DIFFERENTIATION

- On the AP test they will almost always have a Diffy Q Question, where you may have to go backwards from "$\frac{dy}{dx}$" to "$y=$," using the initial condition of when $x=0$, $y=-\ln(4)$. solve the Diffy Q: $y(0)=-\ln(4)$

$$\frac{dy}{dx}=e^{y+x} \rightarrow dy=e^{y}\cdot e^{x}\,dx$$

$$e^{-y}\,dy=e^{x}\,dx \longrightarrow \int e^{-y}\,dy=\int e^{x}\,dx$$

u-sub $u=-y$, $du=-dy$

$$-\int e^{u}\,du=\int e^{x}\,dx \rightarrow -e^{y}=e^{x}+c$$

use initial condition

$$-e^{\ln(4)}=e^{0}+c \longrightarrow -4=1+c \rightarrow -5=c$$

move negative

$$e^{y}=-e^{x}+5 \rightarrow \ln(e^{y})=\ln(5-e^{x})$$

$$y=\ln(5-e^{x})$$

- Training: When you take the derivative of a variable you normally take it w/ respect to "x" but when you take the derivative of a variable different than what the derivative is respect to, Implicit Differentiation must be used.

- Example $\frac{d}{dx}\left[x^{2}+2xy+y^{2}=12\right]$

Take derivative normally for "x" but for "y" you need to multiply by $\frac{dy}{dx}$ after you take derivative normally.

$$2x+2x\cdot\frac{dy}{dx}+2y+2y\cdot\frac{dy}{dx}=0$$

Solve for "$\frac{dy}{dx}$"

$$2x\cdot\frac{dy}{dx}+2y\cdot\frac{dy}{dx}=-2x-2y$$

$$\frac{dy}{dx}=-\frac{2x+2y}{2x+2y}\rightarrow -\frac{1}{1}$$

RIEMANN SUM

X	2	4	7	10	12
f(x)	4	7	13	20	27

- If Asked to Approximate $f'(x)$ use $\frac{rise}{run}$ of two of the closest x-values
- Ex: $f'(3)=?$ $\frac{f(4)-f(2)}{4-2} \longrightarrow \frac{3}{2}\approx f'(3)$

Types of Sums: Kind of self-explanatory

- Left: uses left point for height of sub interval (of each sub-interval)
- Right: uses Right-most point of each sub-interval for height of the sub interval
- Trapezoidal: B_1 is determined by left-most end point, B_2 is determined by right-most end point } For each sub-interval
- Mid-point: Basically uses midpoint between right-most and left-most endpoint of each sub-interval for sub. interval height

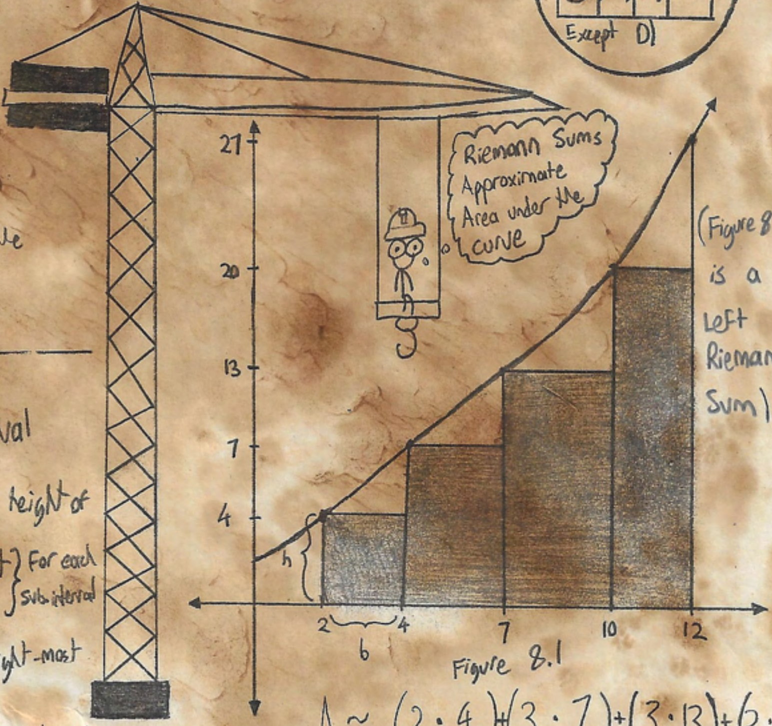


Figure 8.1

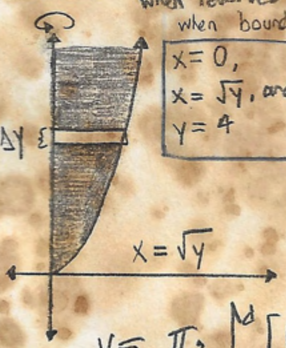
(Figure 8.1 is a Left Riemann Sum)

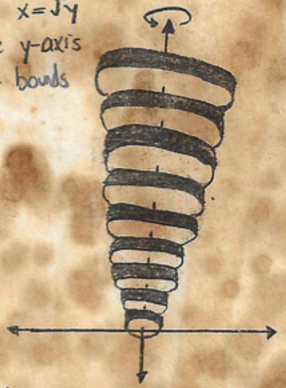
$$A\approx(2\cdot 4)+(3\cdot 7)+(3\cdot 13)+(2\cdot 20)$$

AREA/VOLUME

Volume: What is the volume of $x=\sqrt{y}$ when revolved around the y-axis when bounded by the bounds

$x=0,$
$x=\sqrt{y}$, and
$y=4$





$$V = \pi \cdot \int_c^d [R(y)]^2 \, dy$$

$$V = \pi \cdot \int_0^4 [\sqrt{y}]^2 \, dy = 8\pi$$

If axis of revolution is not touching your function "washer" method needs to be used.
If axis isn't $x=0$ / $y=0$ use T-B or R-L w/ your axis as one of the functions.

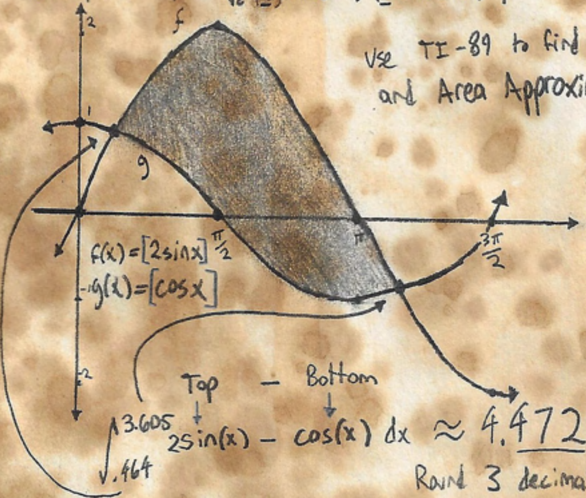
Tools:

- Disc Method: Using ∞ many "discs" to find exact volume of a function revolved around a line.
- Washer Method: when empty space occupies your shape, making donut like washers subtracting the empty space from full shape

Area: when trying to find area between two curves
- Know it will always be +
- Pay close attention to if it's in respect to "x" or to "y"

- In respect to x use $\int_a^b$ (Top function) − (Bottom function) dx
- In respect to y use $\int_c^d$ (Right function) − (left function) dy

Use TI-89 to find intersections and Area Approximation



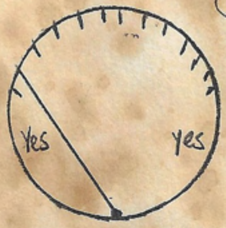
Top − Bottom

$$\int_{.464}^{3.605} 2\sin(x) - \cos(x) \, dx \approx 4.\underline{472}135$$

Round 3 decimal places

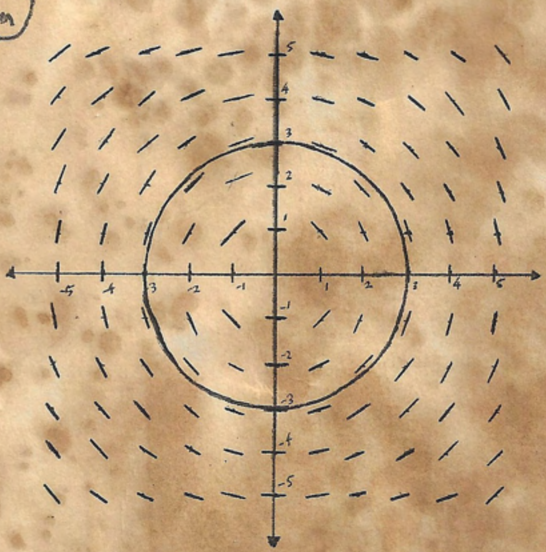
SLOPE Feilds

$$\frac{dy}{dx} = -\frac{x}{y}$$



Slope fields cover every value which "c" could equal when you integrate.

X	Y	dy/dx
0	1	0
1	0	∅
2	3	−2/3
−4	5	4/5
−2	−2	−1
1	−3	1/3
−5	5	1
5	1	−5



- A slope field – created by a differentiable equation – shows the general shape of $\boxed{\text{ALL}}$ possible solutions of a differentiable equation

Sketch: draw a solution curve which passes through (0, 3)

- If you're asked to sketch a solution at a specific coordinate try to follow the directions of the lines.

ESSAY

First and foremost, I'm tired as frick.

My experience in AP Calculus AB has been interesting to say the least, and it's been a wild ride. If you excelled in Pre-Calculus then be sure that everything is going to be okay, and you don't have a lot to worry about. If you follow his rules, and show up to class <u>ON TIME</u>, then this class will be relatively a smooth ride. Most importantly, don't let senioritis get to your head. Too many students screw themselves over in the last parts of the year by habitually being tardy and absent. This really frustrates Mr. Brenneman, seeing his students fall into the pit of not trying enough. I'm very proud to have done well in Calculus, and I've given 100% of my skill to doing well and if I can do it you can aswell. (P.S AP Test Prep is stupid important, pay utmost attention, and study for Quizzes.)

Good Luck