BC Chapter 8 RETAKE

Evaluate each of the following integrals:

1. $\int x\sin x dx$ 2. $\int x^3 \ln x dx$ 3. $\int xe^x dx$ 4. $\int x\sec^2 x dx$ 5. $\int \sqrt{x}\ln x dx$

Evaluate using the method of Partial Fractions:

6. $\int \frac{21}{x^2+7x+10}dx$ 7. $\int \frac{8x-22}{x^2-4x-5}dx$ 8. $\int \frac{11x-15}{x^2-3x+2}dx$

Evaluate using L'Hopital's Rule where appropriate:

9. $\lim_{x\to\infty}\frac{x^3}{e^x}$ 10. $\lim_{x\to 0}\frac{e^x+x-1}{1-e^{-x}}$ 11. $\lim_{x\to\infty}\frac{x^2}{\ln x}$ 12. $\lim_{x\to 0}\frac{x}{1-e^x}$ 13. $\lim_{x\to 0}\frac{6e^{x/6}-(x+2)}{4x}$

Evaluate the Improper Integrals:

14. $\int_0^1 \frac{1}{\sqrt[3]{x}}dx$ 15. $\int_2^{11}(x-3)^{-2/3}dx$ 16. $\int_0^{\infty} e^{-x}dx$ 17. $\int_0^{\infty}\frac{1}{x+1}dx$ 18. $\int_1^{\infty}\frac{1}{(\sqrt{x})^3}dx$

DETAIL $uv - \int v\,du$

Calculus BC Chp. 8 Retake Solutions — Zack Brenneman 8/3/17

① $\int x\sin x\,dx$ | $u = x$, $du = dx$ | $v = -\cos x$, $dv = \sin x\,dx$ | $(x)(-\cos x) - \int(-\cos x)\,dx$

$-x\cos x + \int \cos x\,dx$ | $-x\cos x + \sin x + C$

② $\int x^3 \cdot \ln x\,dx$ | $u = \ln x$, $du = \frac{1}{x}dx$ | $v = x^4/4$, $dv = x^3\,dx$ | $(\ln x)\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4}\cdot\frac{1}{x}\,dx$

$\frac{x^4 \cdot \ln x}{4} - \frac{1}{4}\int x^3\,dx$ | $\frac{x^4 \cdot \ln x}{4} - \frac{1}{4}\cdot\frac{x^4}{4} + C$ | $\frac{x^4 \cdot \ln x}{4} - \frac{x^4}{16} + C$

③ $\int x e^x\,dx$ | $u = x$, $du = dx$ | $v = e^x$, $dv = e^x\,dx$ | $xe^x - \int e^x\,dx$ | $xe^x - e^x + C$

PSST ④ $\int x \cdot \sec^2 x\,dx$ | $u = x$, $du = dx$ | $v = \tan x$, $dv = \sec^2 x\,dx$ | $(x)(\tan x) - \int \tan x\,dx$

side work: $\int \tan x\,dx$ | $-\int \frac{-\sin x}{\cos x}\,dx$ | $u = \cos x$, $du = -\sin x$ | $-\int \frac{1}{u}\,du$ | $-\ln|\cos x| + C$ | $x\tan x + \ln|\cos x| + C$

⑤ $\int x^{1/2} \cdot \ln x\,dx$ | $u = \ln x$, $du = \frac{1}{x}dx$ | $v = \frac{2}{3}\cdot x^{3/2}$, $dv = x^{1/2}\,dx$ | $(\ln x)\left(\frac{2}{3}\cdot x^{3/2}\right) - \int \frac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx$

$\frac{2}{3}x^{3/2}\cdot\ln x - \frac{2}{3}\int x^{1/2}\,dx$ | $\frac{2}{3}\cdot x^{3/2}\cdot\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + C$ | $\frac{2}{3}\cdot x^{3/2}\cdot\ln x - \frac{4}{6}\cdot x^{3/2} + C$

⑥ $\int \frac{21}{x^2 + 7x + 10}\,dx$ | 10: 5, 2; 7 | $\int \frac{21}{(x+2)(x+5)}\,dx$ | $\frac{A}{x+2} + \frac{B}{x+5} = \frac{21}{(x+2)(x+5)}$ | CD = common Denom.

$\frac{A(x+5)}{CD} + \frac{B(x+2)}{CD} = \frac{21}{CD}$ | $A(x+5) + B(x+2) = 21$ Plug in -5 for x to make that thing 0 | $A(-5+5) + B(-5+2) = 21$; $A(0) + B(-3) = 21 \Rightarrow B = -7$

$A(x+5) + B(x+2) = 21$ Plug in -2 for x to make that thing 0. | $A(-2+5) + B(-2+2) = 21$; $A(3) + B(0) = 21$ | $A = 7$ | $\frac{A}{x+2} + \frac{B}{x+5} = \frac{21}{(x+2)(x+5)}$

$\int \frac{7}{x+2}\,dx + \int \frac{-7}{x+5}\,dx$ | $7\cdot\ln|x+2| - 7\cdot\ln|x+5| + C$

⑦ $\int \frac{8x-22}{x^2-4x-5}\,dx$ | $-5,\ 1$; $-5,\ -4$ | $\frac{A}{x-5} + \frac{B}{x+1} = \frac{8x-22}{x^2-4x-5}$ | $A(x+1) + B(x-5) = 8x-22$; $\Sigma_{x=-1}$ $\Sigma_{x=5}$

$A(-1+1) + B(-1-5) = 8(-1) - 22$
$A(0) + B(-6) = -30$ | $B = 5$ | $A(5+1) + B(5-5) = 8(5) - 22$
$A(6) + B(0) = 18$ | $A = 3$

$\int \frac{3}{x-5}\,dx + \int \frac{5}{x+1}\,dx = \int \frac{8x-22}{x^2-4x-5}\,dx$ | $3 \cdot \ln|x-5| + 5 \cdot \ln|x+1| + C$

⑧ $\int \frac{11x-15}{x^2-3x+2}\,dx$ | 2 ; $-2,\ -1$; -3 | $\frac{A}{x-2} + \frac{B}{x-1} = \frac{11x-15}{(x-2)(x-1)}$ | $A(x-1) + B(x-2) = 11x - 15$; $\Sigma_{x=1}$ $\Sigma_{x=2}$

$A(1-1) + B(1-2) = 11(1) - 15$
$A(0) + B(-1) = -4$ | $B = 4$ | $A(2-1) + B(2-2) = 11(2) - 15$
$A(1) + B(0) = 7$ | $A = 7$

$\int \frac{7}{x-2}\,dx + \int \frac{4}{x-1}\,dx = \int \frac{11x-15}{x^2-3x+2}\,dx$ | $7 \cdot \ln|x-2| + 4 \cdot \ln|x-1| + C$

⑨ $\lim_{x\to\infty} \frac{x^3}{e^x}$ | $\frac{\infty^3}{e^\infty} = \frac{\infty}{\infty}$ ✓ | $\lim_{x\to\infty} \frac{3x^2}{e^x} \Rightarrow \lim_{x\to\infty} \frac{6x}{e^x} \Rightarrow \lim_{x\to\infty} \frac{6}{e^x} \Rightarrow \frac{6}{\infty} \Rightarrow 0$

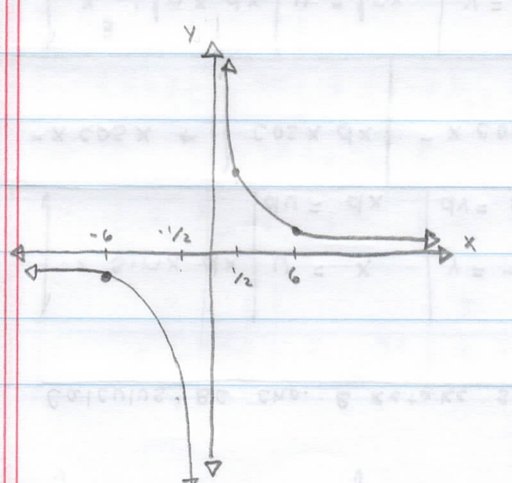
⑩ $\lim_{x\to 0} \frac{e^x + x - 1}{1 - e^{-x}}$ | $\frac{1+0-1}{1-1} = \frac{0}{0}$ ✓ | $\lim_{x\to 0} \frac{e^x + 1}{e^x} \Rightarrow \frac{1+1}{1} \Rightarrow \frac{2}{1} \Rightarrow 2$

⑪ $\lim_{x\to\infty} \frac{x^2}{\ln(x)}$ | $\frac{\infty^2}{\ln(\infty)} = \frac{\infty}{\infty}$ ✓ | $\lim_{x\to\infty} \frac{2x}{1/x} \Rightarrow \frac{2x}{1} \cdot \frac{x}{1} \Rightarrow 2x^2 \Rightarrow 2(\infty)^2 \Rightarrow \infty$

⑫ $\lim_{x\to 0} \frac{x}{1-e^x}$ | $\frac{0}{0}$ ✓ | $\lim_{x\to 0} \frac{1}{-e^x} = \frac{1}{-1} = -1$

Chp. 8 Test Question 13 wont be like this. L'Hopitals Rule will apply for 13.

⑬ $\lim_{x\to 0} \frac{6e^{x/6} - (x+2)}{4x} = \frac{6-2}{0} = \frac{4}{0}$ ✗ | Answer is either $\pm\infty$ or DNE, since there is a V.A. @ $x = 0$



$f(6) = \frac{6e - (8)}{24} \approx \frac{6(2.5) - 8}{24} \approx \frac{5}{24}$

$f(1/2) = \frac{6e^{1/12} - 2.5}{2} \approx \frac{6 - 2.5}{2} \approx \frac{3.5}{2} \approx \frac{7}{4} \approx \frac{42}{28}$

$f(-1/2) = \frac{6e^{-1/12} - (1.5)}{-2} \approx \frac{6 - 1.5}{-2} \approx \frac{4.5}{-2} \approx \frac{-9}{4}$

$f(-6) = \frac{6e^{-1} + 4}{-24} \approx \frac{2+4}{-24} \approx \frac{6}{-24} \approx \frac{-1}{4}$

$\lim_{x\to 0} \frac{6e^{x/6} - (x+2)}{4x} = \boxed{DNE}$

(14) $\int_0^1 \frac{1}{x^{1/3}}\,dx$ | $\int_0^1 x^{-1/3}\,dx$ | $\frac{3}{2}x^{2/3}\Big|_0^1$ | Note: $f(0) = \frac{1}{0}$ = undefined

$\lim_{b\to 0^+}\left[\frac{3}{2}x^{2/3}\right]_b^1$ | $\lim_{b\to 0^+}\left[\frac{3}{2} - \frac{3}{2}b^{2/3}\right]$ | $b \approx +$ small # $-\frac{3}{2}(b)^{2/3} \to 0$ | $\frac{3}{2} - 0$ | $\frac{3}{2}$

(15) $\int_2^{11} (x-3)^{-2/3}\,dx$ | $3(x-3)^{1/3}\Big|_2^{11}$ | Note: $f(3) = \frac{1}{0}$ = undefined

$\lim_{b\to 3^-}\left[3(x-3)^{1/3}\right]_2^b + \lim_{b\to 3^+}\left[3(x-3)^{1/3}\right]_b^{11}$

$3(b-3)^{1/3} - 3(-1) + 3(2) - 3(b-3)^{1/3}$
$3(0) + 3 + 6 - 3(0)$ | $0 + 3 + 6 - 0$ | 9

(16) $\int_0^\infty e^{-x}\,dx$ | $-e^{-x}\Big|_0^\infty$ | $F(\infty) - F(0)$ | $-e^{-\infty} - (-e^0)$ | $\frac{-1}{e^\infty} + 1$ | $0 + 1$ | 1

(17) $\int_0^\infty \frac{1}{x+1}\,dx$ | $\ln|x+1|\Big|_0^\infty$ | $F(\infty) - F(0)$ | $\ln|\infty| - \ln|1|$

$\ln_e|\infty| - \ln_e(1)$ | $(\infty) - (0)$ | ∞

(18) $\int_1^\infty \frac{1}{x^{3/2}}\,dx$ | $\int_1^\infty x^{-3/2}\,dx$ | $\frac{-2}{1}x^{-1/2}\Big|_1^\infty$ | $\frac{-2}{\sqrt{x}}\Big|_1^\infty$ | $F(\infty) - F(1)$

$\frac{-2}{\sqrt{\infty}} - \left(\frac{-2}{\sqrt{1}}\right)$ | $\frac{-2}{\infty} + \frac{2}{1}$ | $0 + 2$ | 2