NO CALCULATOR

Test Form B **Name** ____________ **Date** ____________

Chapter ~~6~~ 7 **Class** ____________ **Section** ____________

1. Determine the area of the region bounded by the graphs of $y = -x^2 + 2x + 3$ and $y = 3$.

(a) $\frac{4}{3}$ (b) $\frac{9}{2}$ (c) $\frac{22}{3}$

(d) $-\frac{4}{3}$ (e) None of these

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$, $x = 2$ and $y = 1$ about the y-axis.

(a) $\frac{93}{5}\pi$ (b) $\frac{120}{7}\pi$ (c) $\frac{47}{5}\pi$

(d) $\frac{62}{5}\pi$ (e) None of these

3. Which of the following integrals represents the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$, $y = 1$ and $x = 2$ about the line $y = 10$?

(a) $\pi\int_1^8 (10 - y)(2 - \sqrt[3]{y})\,dy$ (b) $\pi\int_1^2 [81 - (10 - x^3)^2]\,dx$ (c) $2\pi\int_1^8 y(2 - \sqrt[3]{y})\,dy$

(d) $\pi\int_1^2 [1 - (10 - x^3)^2]\,dx$ (e) None of these

4. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \frac{1}{2}(x - 2)^2$ and $y = 2$ about the y-axis.

(a) $\frac{128\pi}{15}$ (b) $\frac{64\pi}{3}$ (c) $\frac{32\pi}{3}$

(d) $\frac{20\pi}{3}$ (e) None of these

5. Identify the definite integral that represents the arc length of the curve $y = 1/x$ over the interval $[1, 3]$.

(a) $\int_1^3 \sqrt{1 + (\ln x)^2}\,dx$ (b) $\int_1^3 \sqrt{1 + 1/x^2}\,dx$ (c) $\int_1^3 \sqrt{1 + 1/x^4}\,dx$

(d) $\int_1^3 \sqrt{(1/x) + (1/x^4)}\,dx$ (e) None of these

6. Find the volume of the solid with base enclosed by $x^2 + y^2 = 4$ with square cross-sections perpendicular to the x-axis.

Calculator

Test Form C **Name** ______________ **Date** ______________

Chapter ~~6~~ 7 **Class** ______________ **Section** ______________

A graphing calculator/utility is recommended for this test.

1. The integral $\int_a^b [(\sin x + 2) - e^{x^2}]\,dx$ computes the area of a region between two curves. Use a graphing utility to estimate the value of a.

   (a) 0 (b) 1.0 (c) -0.6

   (d) 3.1 (e) None of these

2. Use a graphing utility to graph the region bounded by the graphs of $y = \sqrt{x^3 - x^2}$, $y = 0$, and $x = 3$. Then use calculus to compute the volume of the solid formed by revolving this region about the x-axis.

   (a) $\frac{34\pi}{3}$ (b) $\frac{27\pi}{2}$ (c) 13π

   (d) $\frac{\pi}{12}\left[971 - 216\sqrt{3}\right]$ (e) None of these

3. Use the integration capabilities of a graphing utility to approximate the volume of the solid formed by revolving the region bounded by the graphs of $y = \sin x$ and $y = 0$ in the interval $[0, \pi]$ about the y-axis. Round your answer to three decimal places.

   (a) 30.006 (b) 4.935 (c) 19.739

   (d) 3.142 (e) None of these

4. Use the integration capabilities of a graphing utility to approximate the arc length of the graph of $f(x) = \cos x$ on the interval $[0, \pi]$. Round your answer to three decimal places.

   (a) 2 (b) 3.820 (c) 3.143

   (d) 2.438 (e) None of these

No Calculator

Test Form D **Name** ________________ **Date** __________

Chapter ~~6~~ 7 **Class** ________________ **Section** __________

1. Find the area of the region bounded by the graphs of $y = 1/x$ and $2x + 2y = 5$.

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = e^x, y = 0, x = 0$, and $x = 1$ about the x-axis.

3. Use any ~~the shell~~ method to set up the integral that represents the volume of the solid formed by revolving the region bounded by the graphs of $y = 1/x$ and $2x + 2y = 5$ about the line $y = 1/2$. (Do not evaluate the integral.)

4. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2$ and $y = 4$ about the x-axis.

5. Write the definite integral that represents the arc length of one period of the curve $y = \sin 2x$. (Do not evaluate the integral.)

6. Find the volume of a solid that is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ that has rectangular cross-sections of height 3 that are perpendicular to the x axis.

7. Find the volume of the solid with its base bounded by $x^2 + y^2 = 25$, that has semicircular cross-sections perpendicular to the y axis.

BC Ch 7 Volume RETAKE #2 CALCULATOR Page Name

SKETCH graph, SET UP EQUATION, and SOLVE exact to 3 decimal places.

7. Find the area of the region bounded by the following curves: $y = x^3 - 2x + 1$ and $y = x + 1$.

8. Determine the volume of the solid generated by revolving the plane region bounded by the equations $y = 2^x$ and $y = 3 + \ln x$ about the x-axis.

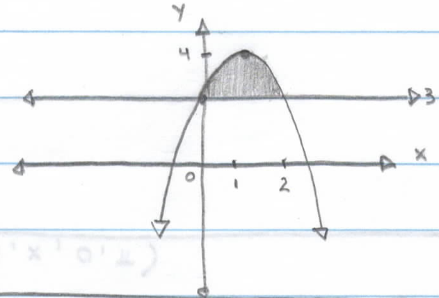
9. Find the volume of the solid generated by revolving the plane region bounded by the equations $y = e^x$, $y = 1$, and $x = 2$ about the line $y = -1$.

10. Determine the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$ with square cross-sections perpendicular to the X-axis.

11. Find the volume of the solid whose base is bounded by $y = x^3$, $y = 0$, and $x = 1$ with a semi-circular cross -sections perpendicular to the X- axis.

Calculus BC Chp. 7 Form B Solutions [No Calculator]

x-coord. of vertex



① $y = -x^2 + 2x + 3$ | $\frac{-b}{2a} = \frac{-2}{-2} = 1$ | $f(1) = -1 + 2 + 3 = 4$

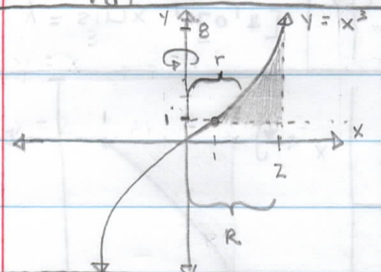
$y = 3$

$3 = -x^2 + 2x + 3$ | $0 = x^2 - 2x$ | $x = 0, 2$

$0 = -x^2 + 2x$ | $0 = x(x-2)$ | Intersection Points

$A = \int_0^2 \left[\left[-x^2 + 2x + 3\right] - \left[3\right]\right] dx$ | $\int_0^2 (-x^2 + 2x)\, dx$ | $\left. \frac{-x^3}{3} + x^2 \right|_0^2$ | $\frac{-8}{3} + \frac{12}{3}$ | $\frac{4}{3}$ | A

②



$y = x^3$ | Outer Radius = 2 | $\pi \int_1^8 \left[(2)^2 - (\sqrt[3]{y})^2\right] dy$ | $y^{2/3}$

$x = \sqrt[3]{y}$ | Inner Radius $= \sqrt[3]{y}$

Disk Method

$\pi \left[4y - \frac{3y^{5/3}}{5}\right]_1^8$ | $\pi \left[\left(32 - \frac{3(32)}{5}\right) - \left(4 - \frac{3}{5}\right)\right]$ | $-96 + 3$

$\pi \left[28 - \frac{93}{5}\right]$ | $\begin{array}{r} \scriptstyle 4 \\ 28 \\ \times\ 5 \\ \hline 140 \end{array}$ | $\pi \left[\frac{140}{5} - \frac{93}{5}\right]$ | $\frac{47\pi}{5}$ | C

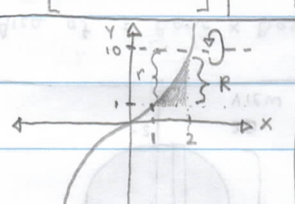
Shell Method

$2\pi \int_a^b p(x) h(x)\, dx$ | $a = 1$, $b = 2$ | $p(x) = x$, $h(x) = x^3 - 1$ | $2\pi \int_1^2 x(x^3 - 1)\, dx$ | $2\pi \int_1^2 (x^4 - x)\, dx$

$2\pi \left. \left(\frac{x^5}{5} - \frac{x^2}{2}\right) \right|_1^2$ | $F(2) - F(1)$ | $2\pi \left[\left(\frac{32}{5} - \frac{4}{2}\right) - \left(\frac{1}{5} - \frac{1}{2}\right)\right]$ | $\begin{array}{r} \scriptstyle 5\ 12 \\ \cancel{62} \\ -\ 15 \\ \hline 47 \end{array}$

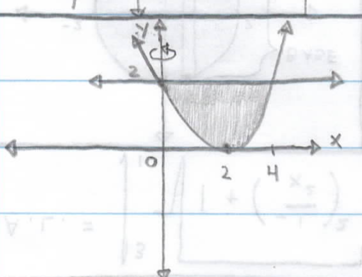
$2\pi \left[\frac{31}{5} - \frac{3}{2}\right]$ | $2\pi \left[\frac{62}{10} - \frac{15}{10}\right]$ | $\cancel{2}\pi \left[\frac{47}{\cancel{10}_5}\right]$ | $\frac{47\pi}{5}$ | I'm more of a disc method guy, but wanted to show both ways in case you prefer shell method.

③



$\pi \int_1^2 \left[(9)^2 - (10 - x^3)^2\right] dx$ | $\pi \int_1^2 \left[81 - (10 - x^3)^2\right] dx$ | B

④



Shell Method : $V = 2\pi \int_a^b p(x) h(x)\, dx$ | $a = 0$, $b = 4$ | $p(x) = x$, $h(x) = 2 - \left(\frac{1}{2}(x-2)^2\right)$

$2\pi \int_0^4 x\left[2 - \frac{1}{2}(x^2 - 4x + 4)\right] dx$ | $2\pi \int_0^4 x\left[\cancel{2} - \frac{x^2}{2} + 2x - \cancel{2}\right] dx$

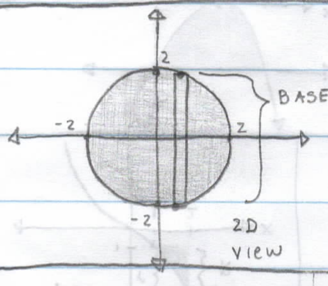
$\begin{array}{r} \scriptstyle 0\,12 \\ \cancel{1}28 \\ -\ 96 \\ \hline 32 \end{array}$

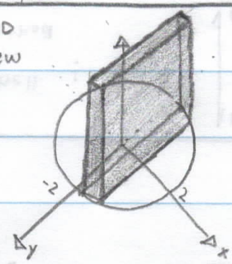
$2\pi \int_0^4 \left(\frac{-x^3}{2} + 2x^2\right) dx$ | $2\pi \left[\frac{-x^4}{8} + \frac{2x^3}{3}\right]_0^4$ | $2\pi \left[-\frac{(16)(\cancel{16})^2}{\cancel{8}_1} + \frac{2(64)}{3}\right]$ | $2\pi \left[-32 + \frac{128}{3}\right]$ | B

$2\pi \left[-96/3 + 128/3\right]$ | $2\pi \left[32/3\right]$ | $64\pi/3$ | B

⑤ Arc Length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$ | $a = 1$, $b = 3$ | $f(x) = 1/x = x^{-1}$ | $f'(x) = -1x^{-2} = -1/x^2$

$A.L. = \int_1^3 \sqrt{1 + \left(\frac{-1}{x^2}\right)^2}\, dx$ | $\int_1^3 \sqrt{1 + \frac{1}{x^4}}\, dx$ | C

⑥ 

3D view 

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

$BASE = \sqrt{4 - x^2} - \left(-\sqrt{4 - x^2}\right)$

$= 2\sqrt{4 - x^2}$

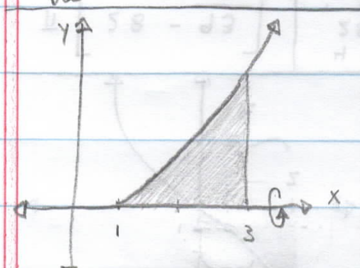
Area of square $=$ Base $\times$ Base $=$ Base^2 | $V = \int_a^b A(x)\, dx$ | $a = -2$, $b = 2$ | $A(x) = \left(2\sqrt{4 - x^2}\right)^2 = 4(4 - x^2) = 16 - 4x^2$

$\int_{-2}^{2} (16 - 4x^2)\, dx$ | $2\int_0^2 (16 - 4x^2)\, dx$ | $2\left(16x - \frac{4x^3}{3}\right)\Big|_0^2$ | $2\left(32 - \frac{32}{3}\right)$ | $2\left(\frac{96}{3} - \frac{32}{3}\right)$ | $\frac{128}{3}$

$\frac{96}{-32}$
64

Calculus BC Chp. 7 Form C Solutions [Calculator Allowed]

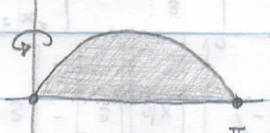
① $\int_a^b \left[(\sin x + 2) - e^{x^2}\right] dx$ | $a \approx -0.601$ | C

② 

$V = \pi \int_1^3 \left[\left(\sqrt{x^3 - x^2}\right)^2\right] dx$ | $\pi \int_1^3 \left[x^3 - x^2\right] dx$

$V = \frac{34\pi}{3}$ | A

③ $y = \sin x$, $[0, \pi]$
$y = 0$, y-axis



$V = 2\pi \int_0^{\pi} x(\sin x)\, dx$ | $V \approx 19.739$

Note: when typing this into calculator, should be
IMPORTANT
$2\pi \int(x \cdot \sin(x), x, 0, \pi)$

④ $A.L. = \int_0^{\pi} \sqrt{1 + (\sin x)^2}\, dx$ | $A.L. \approx 3.820$ | B

Calculus B.C. Chp. 7 Form D Solutions

(1) $2x + 2y = 5$ | $y = \frac{-2x+5}{2}$ | $\frac{1}{x} = \frac{-2x+5}{2}$ | $2 = -2x^2 + 5x$ | $2x^2 - 5x + 2 = 0$

$2y = 5 - 2x$

4×-1, -4, -5

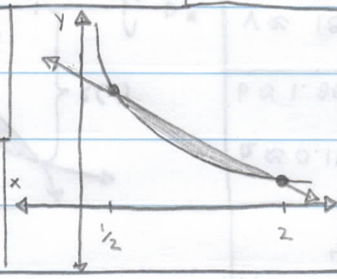
$\underbrace{2x^2 - 4x}\ \underbrace{-x + 2} = 0$

$2x(x-2) - 1(x-2) = 0$

$(2x-1)(x-2) = 0$

$x = \frac{1}{2},\ 2$

$y = -x + 2.5$



$A = \int_{1/2}^{2} \left[\left(-x + \frac{5}{2}\right) - \left(\frac{1}{x}\right)\right] dx$

$\frac{-x^2}{2} + \frac{5}{2}x - \ln|x| \Big|_{1/2}^{2}$ | $F(2) - F(\frac{1}{2})$

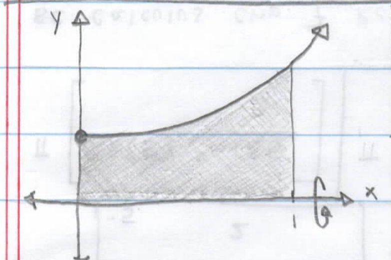
$\left(-2 + 5 - \ln(2)\right) - \left(-\frac{1}{8} + \frac{5}{4} - \ln(\frac{1}{2})\right)$ | $3 - \ln(2) - \left(-\frac{1}{8} + \frac{10}{8} - \ln(\frac{1}{2})\right)$

$3 - \ln(2) - \left(\frac{9}{8} - \ln(\frac{1}{2})\right)$

$\frac{24}{8} - \ln(2) - \frac{9}{8} + \ln(\frac{1}{2})$

$\frac{15}{8} - \ln(2) + \ln(\frac{1}{2})$

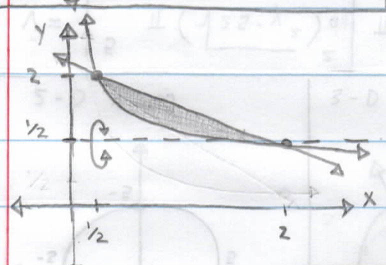
(2)



$V = \pi \int_0^1 (e^x)^2\, dx$ | $\frac{\pi \cdot 1}{2} \int_0^1 2e^{2x}\, dx$ | $u = 2x$, $du = 2\,dx$ | $\frac{\pi}{2}\left(e^{2x}\right)\Big|_0^1$

$\frac{\pi}{2}\left[\left(e^2\right) - \left(e^0\right)\right]$ | $\frac{\pi}{2}\left[e^2 - 1\right]$ | $\frac{\pi(e^2 - 1)}{2}$

(3)

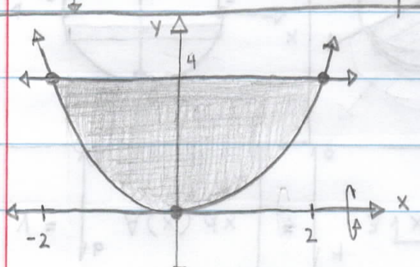


$\frac{1}{1/2} = 2$ | Outer Radius $= (-x + 2.5) - (0.5) = -x + 2$

Inner Radius $= (\frac{1}{x}) - (0.5) = \frac{1}{x} - \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$ | $V = \pi \int_{1/2}^{2} \left[(2 - x)^2 - \left(\frac{1}{x} - \frac{1}{2}\right)^2\right] dx$

(4)



Outer Radius $= 4$

Inner Radius $= x^2$

$V = \pi \int_{-2}^{2} \left[(4)^2 - (x^2)^2\right] dx$

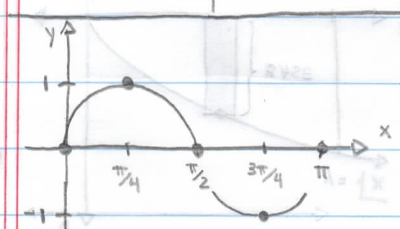
$V = 2\pi \int_0^2 (16 - x^4)\, dx$ | $2\pi\left[16x - \frac{x^5}{5}\right]_0^2$ | $2\pi\left[\frac{160}{5} - \frac{32}{5}\right]$ | $\frac{256\pi}{5}$

128

1
32
$\cdot 5$
160

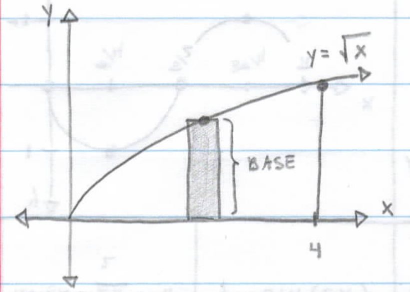
(5) Period $= \frac{2\pi}{2} = \pi$ | $y = \sin(2x)$ | Arc Length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx$ | $a = 0$, $b = \pi$ | $f'(x) = 2\cos(2x)$

(Pretty Cool Question)

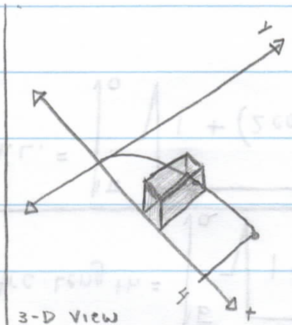


$A.L. = \int_0^{\pi} \sqrt{1 + (2\cos(2x))^2}\, dx$

⑥



2-D View



3-D View

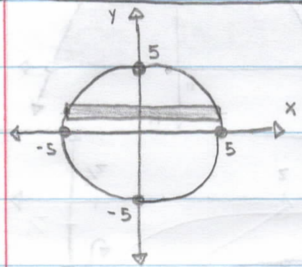
Area of Rectangle = Base · Height

$A_{R.} = (\sqrt{x}) \cdot (3)$

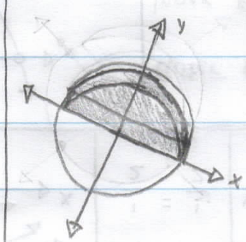
Base Height

$V = \int_a^b A(x)\,dx = \int_0^4 3\sqrt{x}\,dx \quad \int_0^4 3x^{1/2}\,dx \quad \left.\frac{3 \cdot 2x^{3/2}}{3}\right|_0^4 \quad 2(4)^{3/2} \quad 2(2)^3 \quad 2(8) \quad 16$

⑦



2-D View



3-D View

Area of Semi-Circle $= \frac{\pi r^2}{2}$ $\quad x^2 + y^2 = 25 \quad x^2 = 25 - y^2 \quad x = \pm\sqrt{25 - y^2}$

Base $= \sqrt{25-y^2} - \left(-\sqrt{25-y^2}\right)$

$= 2\sqrt{25-y^2}$

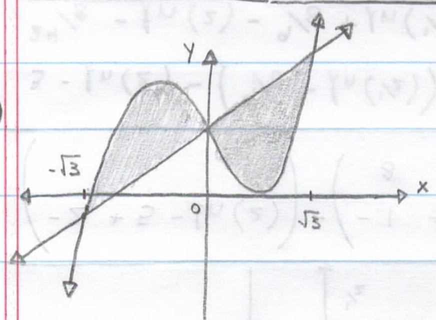
Radius $= \left(2\sqrt{25-y^2}\right) \div 2 = \sqrt{25-y^2}$

$V = \int_{-5}^{5} \frac{\pi\left(\sqrt{25-y^2}\right)^2}{2}\,dy \quad \frac{\pi}{2}\int_{-5}^{5} (25-y^2)\,dy \quad \pi\int_0^5 (25-y^2)\,dy \quad \pi\left[25y - \frac{y^3}{3}\right]_0^5$

$\pi\left[25(5) - \frac{125}{3}\right] \quad \pi\left[125 - \frac{125}{3}\right] \quad \pi\left[\frac{375}{3} - \frac{125}{3}\right] \quad \pi\left[\frac{250}{3}\right] \quad \frac{250\pi}{3}$

BC Calculus Chp. 7 Retake # 2 Solutions

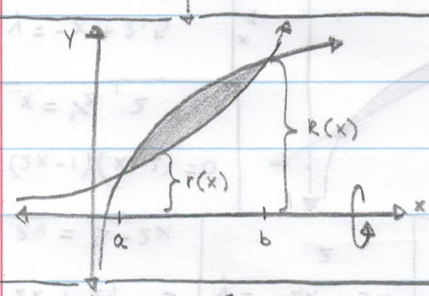
⑦



$A = \int_{-\sqrt{3}}^{0} \left((x^3 - 2x + 1) - (x+1)\right)dx + \int_0^{\sqrt{3}} \left((x+1) - (x^3 - 2x + 1)\right)dx$

$A = \frac{9}{4} + \frac{9}{4} = \frac{18}{4} = \frac{9}{2} = 4.5$

⑧



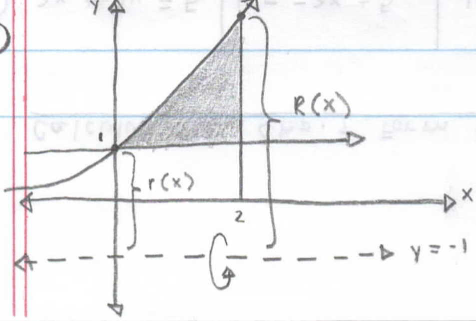
$a \approx 0.151$

$b \approx 1.855$

$V \approx \pi \int_a^b \left[\left(3 + \ln x\right)^2 - \left(2^x\right)^2\right]dx$

$V \approx 18.424$

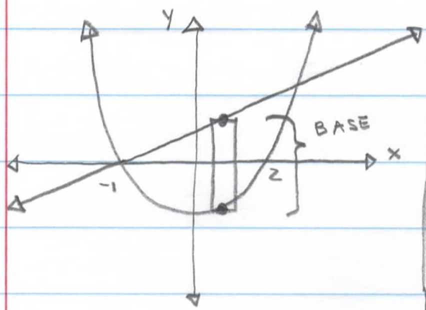
⑨



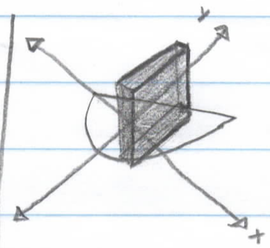
$V = \pi \int_0^2 \left[\left(1 + e^x\right)^2 - (2)^2\right]dx$

$V \approx 105.486$

10



2D - View



3D - View

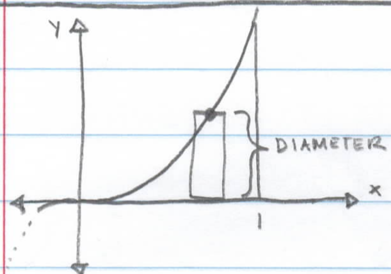
$A_s = b \cdot h = b \cdot b = b^2$

base $= (x+1) - (x^2-1)$

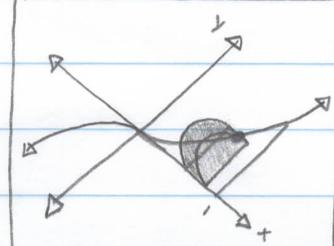
$$V = \int_{-1}^{2} \left[(x+1) - (x^2-1)\right]^2 dx$$

$V = 81/10$

11



2D - View



3D - View

$A_{s.s.} = \dfrac{\pi r^2}{2}$

Diameter $= x^3$

Radius $= x^3/2$

$$V = \int_0^1 \left(\frac{\pi \left(x^3/2\right)^2}{2}\right) dx = \frac{\pi}{2}\int_0^1 \frac{x^6}{4}\, dx$$

$V \approx \pi/56$