Calculus BC Chapter 6 Part 1 Study Guide [Calculator Allowed]

1. A mold culture triples its mass every four days. Find the growth model for a plate seeded with 1.1 grams of mold.

2. The population P of a town is given by $P = 1{,}000e^{kt}$. Let $t = 0$ correspond to the year 1870 and suppose the population in 1850 was 700. Find the value of k (to 3 decimal places) and then predict the population in 1900.

3. Write an equation for the amount Q of radio-active substance with a half-life of 30 days, if 14 grams are present when $t = 0$.

4. The number of fruit flies increases according to the law of exponential growth. If initially there are 16 fruit flies and after 8 hours there are 42, find the number of fruit flies after t hours.

5. A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the environment can support no more than 4,000 elk.

   a) Write a logistic model for the number of elk in terms of t.
   b) Use the model to estimate the number of elk in 15 years.

6. Use Euler's Method to approximate the particular solution $y(0.4)$ of the differential equation $\frac{dy}{dx} = x - y$ passing through the point $(0, 1)$ using 4 steps with $h = 0.1$.

Calculus BC Chapter 6 Part 2 Study Guide [No Calculator]

1. Solve the differential equation $xy' = y$

2. The rate of change of y w/ respect to x is inversely proportional to the cube root of y.

   a. Write a differential equation for the given statement.

   b. Solve the differential equation in part a.

3. Find the function $y = f(x)$ passing through the point $(1,1)$ that has the first derivative $y' = 2xy$

4. Find the particular solution to the differential equation $y' = \cos x$ given the general solution $y = \sin x + C$ & the initial condition $y(\pi) = 1$.

5. Find the particular solution to the differential equation $y' = 5y$ given the general solution $y = Ce^{5x}$ & the initial condition $y(1) = 30$.

6. Use integration to find a general solution to the differential equation:

$$\frac{dy}{dx} = \frac{4x}{1-2x^2}$$

7. Find the general solution to the first-order differential equation:

$$(4-x)\,dy + 2y\,dx = 0$$

8. Find the general solution to the first order differential equation:

$$y \cdot y' - (x+4) = 0$$

9. Find the general solution to the differential equation:

$$e^{2y} \cdot y' = x^3$$

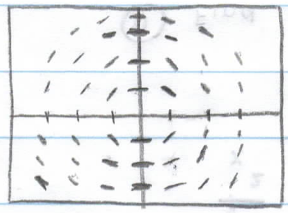
10. Find the general solution to the differential equation:

$$\frac{y'}{x^2} = \frac{x^3}{y^2}$$

11. Find the particular solution to the differential equation $y' = 40 - y$ that satisfies the initial condition $y(0) = 5$.

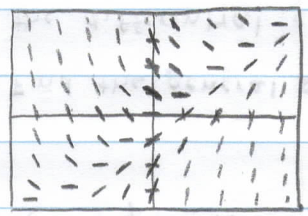
(12) - (17) These questions ask you to match the differential equation to its slope field, so for example:

(12) $\frac{dy}{dx} = x - y$: ______



A

(13) $\frac{dy}{dx} = -\frac{x}{y}$: ______



B

(18) - (19) These questions ask you to draw the slope field for the differential equation:

(18) $f'(x) = x + 1$, (19) is similar ☺

Calculus BC Chapter 6 Part 1 S.G. Solutions [Calculator Allowed]

① $y = Ce^{kt}$

C = initial value = 1.1 grams

$y = 3.3$, $t = 4$

Note: t = time = 4 days

y = Amount after specific time interval

$3.3 = 1.1e^{k(4)}$

$3 = e^{4k}$

$\ln(3) = \ln(e^{4k})$

$\ln(3) = 4k \cdot \ln(e)$

$\ln(3) = 4k$

$k \approx 0.275$

$y = 1.1e^{0.275t}$ where t is time in days.

② $P = 1{,}000e^{kt}$

Note: $t = 0 \Rightarrow 1870$, then $t = -20 \Rightarrow 1850$ & $t = 30 \Rightarrow 1900$

$P = 1{,}000e^{kt}$

$700 = 1{,}000e^{k(-20)}$

$0.7 = e^{-k(20)}$

$\ln(0.7) = \ln(e^{-20k})$

$\ln(0.7) = -20k$

$k \approx 0.018$

$P = 1{,}000e^{0.018(30)}$

$P \approx 1716.007$ ← Approximate population in 1900.

③ $Q = Ce^{kt}$

C = Initial value = 14 grams

$Q = 7$ grams, $t = 30$

Note: $t=0$ gives us Initial Amount, but since substance has a half life of 30 days, then when $t = 30$, Q = Half of 14 which is 7.

$7 = 14e^{30k}$

$0.5 = e^{30k}$

$\ln(0.5) = \ln(e^{30k})$

$\ln(0.5) = 30k$

$k \approx -0.023$

$Q = 14e^{-0.023t}$

④ $y = Ce^{kt}$

C = Initial Value = 16

$y = 42$; $t = 8$

$42 = 16e^{8k}$

$2.625 = e^{8k}$

$\ln(2.625) = 8k$

$k = \dfrac{\ln(2.625)}{8}$

$y = 16e^{t\left(\frac{\ln(2.625)}{8}\right)}$

⑤ $P = \dfrac{L}{1 + be^{-kt}}$

Note: P = Population After Specific Time

L = Carrying Capacity (Upper Limit)

t = time

k = growth / decay rate

$b = \pm e^{-C}$. Don't worry too much about it. It's from derivation.

$P(0) = \dfrac{4{,}000}{1 + be^{-k(0)}}$

$P(0) = 40$

$40 = \dfrac{4{,}000}{1 + be^{0}}$

$\dfrac{40}{1} = \dfrac{4{,}000}{1+b}$

$40(1+b) = 4{,}000$

$1 + b = 100$

$b = 99$

$P(5) = 104$

$104 = \dfrac{4{,}000}{1 + 99e^{-5k}}$

$104(1 + 99e^{-5k}) = 4{,}000$

$1 + 99e^{-5k} = \dfrac{500}{13}$

$99e^{-5k} = \dfrac{487}{13}$

$e^{-5k} = \dfrac{487}{13} \cdot \dfrac{1}{99}$

$e^{-5k} = \dfrac{487}{1287}$

$\ln(e^{-5k}) = \ln\left(\dfrac{487}{1287}\right)$

$-5k = \ln\left(\dfrac{487}{1287}\right)$

$k \approx 0.194$

Note: Round once you solve for k. Don't round early, b/c it will screw up your k-value.

a) $P = \dfrac{4{,}000}{1 + 99e^{-0.194t}}$

b) $P(15) = \dfrac{4{,}000}{1 + 99e^{-0.194(15)}}$

$P(15) \approx 625.675$ elk

⑥ $h = 0.1$, $x_0 = 0$ | $y_0 = 1$ | $\dfrac{dy}{dx} = F(x, y) = x - y$ | Note: $x_1 = 0.1$, $x_2 = 0.2$ | $x_3 = 0.3$

Step 1: $y_1 = y(0.1) = y_0 + hF(x_0, y_0) = 1 + (0.1)(0 - 1) = 1 + (-0.1) = 0.9$

Step 2: $y_2 = y(0.2) = y_1 + hF(x_1, y_1) = 0.9 + (0.1)(0.1 - 0.9) = 0.9 - 0.08 = 0.82$

Step 3: $y_3 = y(0.3) = y_2 + hF(x_2, y_2) = 0.82 + (0.1)(0.2 - 0.82) = 0.82 - 0.062 = 0.758$

Step 4: $y_4 = y(0.4) = y_3 + hF(x_3, y_3) = 0.758 + (0.1)(0.3 - 0.758) = 0.758 - 0.0458 = 0.712$

Chapter 6 BC Part 2 S.G. Solutions [No Calculator]

① $xy' = y$ | $x \cdot \dfrac{dy}{dx} = y$ | $x\,dy = y\,dx$ | $\dfrac{1}{y}\,dy = \dfrac{1}{x}\,dx$ | $\displaystyle\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\ln|y| = \ln|x| + c$ | $e^{\ln|y|} = e^{\ln|x| + c}$ | $e^{\ln|y|} = e^{\ln|x|} \cdot e^{c}$

$y = x \cdot e^{c}$ | $y = x \cdot C$ | $y = Cx$

② a. $\dfrac{dy}{dx} = \dfrac{k}{\sqrt[3]{y}}$ | $y^{1/3}\,dy = k\,dx$ | $\displaystyle\int y^{1/3}\,dy = \int k\,dx$ | $\dfrac{3}{4}y^{4/3} = kx + C$

$y^{4/3} = \dfrac{4}{3}(kx + C)$ | $y^{4/3} = \dfrac{4}{3}kx + C$ | $y = \left(\dfrac{4}{3}kx + C\right)^{3/4}$

③ $\dfrac{dy}{dx} = 2xy$ | $\dfrac{1}{y}\,dy = 2x\,dx$ | $\displaystyle\int \frac{1}{y}\,dy = \int 2x\,dx$ | $\ln|y| = x^2 + c$

$e^{\ln|y|} = e^{x^2} \cdot e^{c}$ | $y = Ce^{x^2}$ | $1 = Ce^{(1)^2}$ | $1 = Ce$, $C = \dfrac{1}{e}$ | $y = \dfrac{e^{x^2}}{e}$

$y = e^{x^2 - 1}$ | check: $y' = e^{x^2 - 1} \cdot 2x$ | $y'(1) = e^{0} \cdot 2(1) = 2$ ✓, $\dfrac{dy}{dx}(1,1) = 2(1)(1) = 2$

(-1,0)

④ $y' = \cos x$, $y = \sin x + C$ | $y(\pi) = 1$ | $1 = \sin(\pi) + C$, $1 = 0 + C$ | $y = \sin x + 1$

⑤ $y' = 5y$, $y = Ce^{5x}$ | $y(1) = 30$ | $30 = Ce^{5(1)}$, $30 = Ce^{5}$ | $C = \dfrac{30}{e^5}$ | $y = \dfrac{30e^{5x}}{e^5}$ | $y = 30e^{5x - 5}$

(6) $\frac{dy}{dx} = \frac{4x}{1-2x^2}$ | $dy = \frac{4x}{1-2x^2}\,dx$ | $\int dy = -\int \frac{-4x}{1-2x^2}\,dx$ | $u = 1-2x^2$, $du = -4x\,dx$

$y = -\int \frac{1}{u}\,du$ | $y = -\ln|u| + c$ | $y = -\ln|1-2x^2| + c$

(7) $(4-x)\,dy + 2y\,dx = 0$
$(4-x)\,dy = -2y\,dx$ | $\frac{-1}{2y}\,dy = \frac{1}{4-x}\,dx$ | $\frac{-1}{2}\int \frac{-1 \cdot 2}{2y}\,dy = -\int \frac{-1}{4-x}\,dx$

$u = 2y$, $du = 2\,dy$ | $v = 4-x$, $dv = -1\,dx$ | $\frac{-1}{2}\int \frac{1}{u}\,du = -\int \frac{1}{v}\,dv$ | $\frac{-1}{2} \cdot \ln|u| = -\ln|v| + c$

$\frac{-1}{2} \cdot \ln|2y| = -\ln|4-x| + c$ | $c = \ln|4-x| - \frac{\ln|2y|}{2}$ | FYI, this question on the test is Multiple Choice.

(8) $y \cdot y' - (x+4) = 0$ | $y \cdot \frac{dy}{dx} = x+4$ | $y \cdot dy = (x+4)\,dx$

$\int y\,dy = \int (x+4)\,dx$ | $\frac{y^2}{2} = \frac{x^2}{2} + 4x + c$ | $y^2 = x^2 + 8x + C$, $C = y^2 - x^2 - 8x$

(9) $e^{2y} \cdot \frac{dy}{dx} = x^3$ | $e^{2y}\,dy = x^3\,dx$ | $\frac{1}{2}\int 2e^{2y}\,dy = \int x^3\,dx$ | $u = 2y$, $du = 2\,dy$

$\frac{1}{2}\int e^u\,du = \int x^3\,dx$ | $\frac{1}{2} \cdot e^{2y} = \frac{x^4}{4} + c$ | $e^{2y} = \frac{x^4}{2} + c$

$\ln(e^{2y}) = \ln(x^4/2 + c)$ | $2y = \ln\left(\frac{x^4}{2} + c\right)$ | $y = \frac{1}{2} \cdot \ln\left(\frac{x^4}{2} + c\right)$

(10) $\frac{y'}{x^2} = \frac{x^3}{y^2}$ | $y^2 \cdot y' = x^3 \cdot x^2$ | $y^2 \cdot \frac{dy}{dx} = x^5$ | $y^2\,dy = x^5\,dx$

$\int y^2\,dy = \int x^5\,dx$ | $\left(\frac{y^3}{3}\right) = \left(\frac{x^6}{6} + c\right)$ | $2y^3 = x^6 + c$

(11) $\frac{dy}{dx} = 40 - y$ | $dy = (40-y)\,dx$ | $\frac{1}{(40-y)}\,dy = dx$ | $-\int \frac{-1}{40-y}\,dy = \int dx$

$u = 40-y$, $du = -1\,dy$ | $-\int \frac{1}{u}\,du = \int dx$ | $-\ln|40-y| = x + c$, $\ln|40-y| = c - x$ | $e^{\ln|40-y|} = e^{c-x}$