Calculus BC Retake Ch 5.4-5.7

1. Differentiate with respect to x: $y = e^{7-(5/x)}$.

   [A] $\frac{dy}{dx} = e^{7-(5/x)}$ [B] $\frac{dy}{dx} = e^{5/x^2}$ [C] $\frac{dy}{dx} = \frac{5}{x^2} e^{7-(5/x)}$

   [D] $\frac{dy}{dx} = -e^{7-(5/x)}$ [E] None of these

2. Find y': $y = e^{\sec x}$.

3. Differentiate: $f(x) = \frac{1}{(4+e^{2x})^4}$.

4. Find y' if $y = e^{\cot x^2}$

   [A] $e^{-2x \csc^2 x^2}$ [B] $-2x \csc^2 x^2 \, e^{\cot x^2}$ [C] $(\cot x^2) \, e^{\cot x^2 - 1}$

   [D] $-\csc^2 x^2 \, e^{\cot x^2}$ [E] None of these

5. Find y' if $y = e^{\sin\sqrt{x}}$.

   [A] $\left(\cos\sqrt{x}\right)e^{\sin\sqrt{x}}$ [B] $\left(\sin\sqrt{x}\right)e^{\sin\sqrt{x}-1}$ [C] $\frac{e^{\cos\sqrt{x}}}{2\sqrt{x}}$

   [D] $\frac{\cos\sqrt{x}}{2\sqrt{x}} e^{\sin\sqrt{x}}$ [E] None of these

6. Find y': $y = e^{1/x}$.

7. Find $\frac{dy}{dx}$ if $ye^x - x = y^2$.

8. Find the slope of the tangent line to the graph of $y = \ln(xe^x)$ at the point where $x = 3$.

   [A] $e^3 + \ln 3$ [B] $\frac{4}{3}$ [C] $\ln 3 + 3$ [D] $\frac{1}{3}$ [E] None of these

9. Evaluate the integral: $\int \sin x \, e^{\cos x} \, dx$.

10. Evaluate the integral: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$.

[A] $\sqrt{x}\, e^{\sqrt{x}} + C$ [B] $2e^{\sqrt{x}} + C$ [C] $\sqrt{x}\, e^{\sqrt{x}+1} + C$

[D] $\frac{1}{2} e^{\sqrt{x}} + C$ [E] None of these

11. Evaluate the indefinite integral: $\int \frac{1}{x^2 e^{3/x}} \, dx$.

12. Evaluate the integral: $\int \frac{e^x}{\sqrt{e^x + 1}} \, dx$.

13. Evaluate the integral: $\int e^{(ax+b)} \, dx$.

[A] $\frac{1}{a} e^{(ax+b)} + C$ [B] $ae^{(ax+b)} + C$ [C] $e^{(ax+b)} + C$

[D] $e^{(ax^2/2+bx+c)}$ [E] None of these

14. Find $\frac{dy}{dx}$ if $y = 3^x x^3$.

[A] $9x^2$ [B] $3^{x-1}x^2[9 + x^2]$ [C] $2x^2 3^{x-1}$

[D] $3^x x^2[3 + (\ln 3)x]$ [E] None of these

15. Find $\frac{dy}{dx}$ if $y = x^e e^x$.

16. Differentiate: $y = x^{e^x}$.

[A] $e^x x^{e^{x-1}}$ [B] $xe^x + e^x$ [C] e^x [D] $x^{e^x}\left[\frac{e^x}{x} + (\ln x)(e^x)\right]$ [E] None of these

17. Find the derivative: $y = \arctan e^x$.

18. Find $\frac{dy}{dx}$ for $y = \arctan\frac{x}{2}$.

[A] $\frac{1}{\sqrt{4-x^2}}$ [B] $\frac{4}{1+x^2}$ [C] $\frac{2}{4+x^2}$ [D] $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ [E] $\frac{4}{4+x^2}$

19. Evaluate the integral: $\int \frac{\cos x}{9+\sin^2 x}\,dx.$

20. Evaluate the integral: $\int \frac{x+2}{\sqrt{4-x^2}}\,dx.$

[A] $-\frac{1}{2}\sqrt{4-x^2}+2\arcsin\frac{x}{2}+C$ [B] $\ln|2-x|+C$

[C] $-\sqrt{4-x^2}+2\arcsin\frac{x}{2}+C$ [D] $x^2+2x+\arcsin\frac{x}{2}+C$

[E] None of these

21. Evaluate the integral: $\int \frac{5x+16}{x^2+9}\,dx.$

22. Evaluate the integral: $\int \frac{x}{81+x^4}\,dx.$

[A] $\frac{1}{18}\arctan\frac{x^2}{9}+C$ [B] $\frac{1}{2}\arcsin\frac{x^2}{9}+C$ [C] $\frac{1}{9}\arctan\frac{x^2}{9}+C$

[D] $\frac{1}{18}\operatorname{arcsec}\frac{x^2}{9}+C$ [E] None of these

23. Evaluate the integral: $\int \frac{dx}{\sqrt{8+2x-x^2}}.$

[A] $\frac{1}{3}\operatorname{arcsec}\frac{x-1}{3}+C$ [B] $\ln\sqrt{8+2x-x^2}$ [C] $\arcsin\frac{x-1}{3}+C$

[D] $\sqrt{8+2x-x^2}+C$ [E] None of these

24. Evaluate the integral: $\int_1^4 \frac{1}{x^2-2x+10}\,dx$.

[A] -0.0419 [B] 1.249 [C] $-\frac{\pi}{4}$ [D] $\frac{\pi}{12}$ [E] None of these

25. Solve the differential equation: $\sqrt{9-x^2}\,y' = 3$.

[A] $y = 3\arcsin\frac{x}{3} + C$ [B] $y = -\frac{3}{x}\sqrt{9-x^2} + C$ [C] $y = 6\sqrt{9-x^2} + C$

[D] $y = \arcsin\frac{x}{3} + C$ [E] None of these

AP Calculus BC Retake Chp. 5.4 - 5.7

(1) $y = e^{7 - 5/x}$ | $y' = e^{7-5/x} \cdot (5x^{-2})$ | $y' = \frac{5}{x^2} \cdot e^{7-5/x}$ | C

(2) $y = e^{\sec x}$ | PRST | $y' = e^{\sec x} \cdot \sec x \cdot \tan x$

(3) $f(x) = \frac{1}{(4+e^{2x})^4}$ | $f(x) = (4+e^{2x})^{-4}$ | $f'(x) = -4(4+e^{2x})^{-5} \cdot (2e^{2x})$ $= -8e^{2x}(4+e^{2x})^{-5}$

(4) $y = e^{\cot(x^2)}$ | MCSCSC4 | $y' = e^{\cot(x^2)} \cdot (-\csc^2(x^2)) \cdot 2x$ | $y' = -2x \csc^2(x^2) \cdot e^{\cot(x^2)}$ | B

(5) $y = e^{\sin(\sqrt{x})}$ | $y' = e^{\sin(\sqrt{x})} \cdot \cos(\sqrt{x}) \cdot (\frac{1}{2}x^{-1/2})$ | $y' = \frac{\cos(\sqrt{x}) \cdot e^{\sin(\sqrt{x})}}{2\sqrt{x}}$ | D

(6) $y = e^{1/x}$ (x^{-1}) | $y' = e^{1/x} \cdot (-1x^{-2})$ | $y' = \frac{-e^{1/x}}{x^2}$

(7) $ye^x - x = y^2$ | $y \cdot e^x + e^x \cdot dy/dx - 1 = 2y \cdot dy/dx$ (F s' + s F') | $e^x \cdot dy/dx - 2y \cdot dy/dx = 1 - ye^x$

$\frac{dy}{dx}(e^x - 2y) = 1 - ye^x$ | $\frac{dy}{dx} = \frac{1 - ye^x}{e^x - 2y}$

(8) $y = \ln(xe^x)$ @ $x = 3$ | $y' = \frac{1}{xe^x} \cdot (x)(e^x) + (e^x)(1)$ | $y' = \frac{e^x(x+1)}{e^x(x)}$ | $y'(3) = \frac{4}{3}$ | B

(9) $-\int -\sin x \cdot e^{\cos x}\,dx$ | $u = \cos x$, $du = -\sin x\,dx$ | $-\int e^u\,du$ | $-e^u + c$ | $-e^{\cos x} + c$

(10) $2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$ | $u = \sqrt{x}$, $du = \frac{1}{2}x^{-1/2}dx$ | $2\int e^u\,du$ | $2e^{\sqrt{x}} + c$ | B

(11) $\frac{1}{3}\int \frac{3 \cdot 1}{x^2 e^{3/x}}\,dx$ | $\int x^{-2} e^{-3/x}\,dx$ | $u = -3/x = -3x^{-1}$, $du = 3x^{-2}dx = 3/x^2\,dx$ | $\frac{1}{3}\int e^u\,du$ | $\frac{1}{3} \cdot e^{-3/x} + c$

(12) $\int \frac{e^x}{\sqrt{e^x+1}}\,dx$ | $u = e^x + 1$, $du = e^x dx$ | $\int \frac{1}{u^{1/2}}\,du$ | $\int u^{-1/2}\,du$ | $\frac{2}{1}u^{1/2} + c$

$2(e^x+1)^{1/2} + c$

(13) $\int e^{ax+b}\,dx$ | $u = ax+b$, $du = a\,dx$ | $\frac{1}{a}\int e^{u}\cdot du$ | $\frac{1}{a}\cdot e^{ax+b} + C$ | A

(14) $y = 3^x \cdot x^3$ | $y' = (3^x)(3x^2) + (x^3)\cdot \ln(3)\cdot 3^x$ | $y' = 3^x x^2 (3 + x\ln(3))$ | D

$e^x = \ln e \cdot e^x$ | F S' S F'

(15) $y = x^e e^x$ | $y' = (x^e)(e^x) + (e^x)(ex^{e-1})$ | $y' = e^x(x^e + ex^{e-1})$

F S' + S F'

(16) $y = x^{e^x}$ | $\ln y = \ln(x^{e^x})$ | $\ln y = e^x \cdot \ln x$ | $\frac{1}{y}\cdot\frac{dy}{dx} = (e^x)\left(\frac{1}{x}\right) + (\ln x)(e^x)$

F S' S F'

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{e^x}{x} + e^x\cdot\ln x$ | $\frac{dy}{dx} = \left[\frac{e^x}{x} + (\ln x)(e^x)\right]\cdot x^{e^x}$ | D

(17) $y = \arctan e^x$ | $u = e^x$, $du = e^x$ | $y' = \frac{1}{1+e^{2x}}\cdot e^x$ | $y' = \frac{e^x}{1+e^{2x}}$

(18) $y = \arctan(x/2)$ | $y' = \frac{1}{1+(x/2)^2}\cdot\frac{1}{2}$ | $y' = \frac{1}{1+(x^2/4)}\cdot\frac{1}{2}$

$y' = \frac{1}{2+x^2/2}$ | $y' = \frac{1}{\frac{4+x^2}{2}}$ | $y' = \frac{1}{1}\cdot\frac{2}{4+x^2}$ | $y' = \frac{2}{4+x^2}$ | C

(19) $\int \frac{\cos x}{9+\sin^2 x}\,dx$ | $u = \sin x$, $du = \cos x\,dx$ | $\int \frac{1}{9+u^2}\,du$ | $a^2 = 9$, $a = 3$

$\int \frac{1}{a^2+u^2}\,du$ | $\frac{1}{3}\arctan(u/a) + C$ | $\frac{1}{3}\arctan\left(\frac{\sin x}{3}\right) + C$

(20) $\int \frac{x+2}{\sqrt{4-x^2}}\,dx$ | $\frac{-1}{2}\int \frac{-2x}{\sqrt{4-x^2}}\,dx + 2\int \frac{1}{\sqrt{4-x^2}}\,dx$ | $u = 4-x^2$, $du = -2x\,dx$ | $v = x$, $a = 2$ | $dv = dx$

$-\frac{1}{2}\int u^{-1/2}\,du + 2\int \frac{1}{\sqrt{a^2-v^2}}\,dv$ | $-\frac{1\cdot 2}{2}\cdot u^{1/2} + 2\cdot\arctan\left(\frac{v}{a}\right) + C$

$-(4-x^2)^{1/2} + 2\arctan\left(\frac{x}{2}\right) + C$ | C

(21) $\int \frac{5x+16}{x^2+9}\,dx$ | $\frac{5}{2}\int \frac{2x}{x^2+9}\,dx + 16\int \frac{1}{x^2+9}\,dx$ | $u = x^2+9$, $du = 2x\,dx$ | $a = 3$, $v = x$ | $dv = dx$

$\frac{5}{2}\int \frac{1}{u}\,du + 16\int \frac{1}{v^2+a^2}\,dv$ | $\frac{5}{2}\cdot\ln|x^2+9| + \frac{16}{3}\arctan\left(\frac{x}{3}\right) + C$

(22) $\frac{1}{2}\int \frac{2x}{81+x^4}\,dx$ | $u = x^2$, $a = 9$ | $du = 2x\,dx$ | $\frac{1}{2}\int \frac{1}{a^2+u^2}\,du$ | $\frac{1}{18}\cdot\arctan\left(\frac{x^2}{9}\right)+C$ | A

(23) $\int \frac{dx}{\sqrt{8+2x-x^2}}$ | $-x^2+2x+8$ | -8, 4, -2, 2 | $-x^2-2x+4x+8$ | $-x(x+2)+4(x+2)$

$(x+2)(4-x)$ | $\int \frac{dx}{\sqrt{(x+2)(4-x)}}$ | NOT GUNNA WORK... NEW APPROACH

$8+2x-x^2 = -(x^2-2x+8)$ $= -(x-1)^2+9$ | $\int \frac{dx}{\sqrt{9-(x-1)^2}}$ | $u = x-1$, $a = 3$ | $du = dx$

$\int \frac{1}{\sqrt{a^2-u^2}}\,du$ | $\arcsin\left(\frac{u}{a}\right)+C$ | $\arcsin\left(\frac{x-1}{3}\right)+C$ | C

(24) $\int_1^4 \frac{1}{x^2-2x+10}\,dx$ | $x^2-2x+10 = (x-1)^2+9$ | $\int_1^4 \frac{1}{9+(x-1)^2}\,dx$

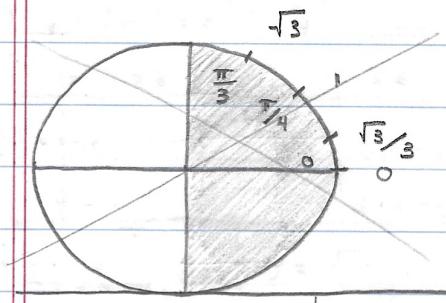
$u = x-1$, $du = dx$ | $a = 3$ | $\int_1^4 \frac{1}{a^2+u^2}\,du$ | $\frac{1}{3}\arctan\left(\frac{x-1}{3}\right)\Big|_1^4$ | $F(4)-F(1)$

$\frac{1}{3}(\arctan(3/3)-\arctan(0))$

$\frac{1}{3}(\arctan(1)-\arctan(0))$

$\frac{1}{3}((\pi/4)-0) = \pi/12$

Note: arctan(x) reads, "what angle in the unit circle gives me a tangent value of x?"



$= \frac{\pi}{12}$ D

(25) $\sqrt{9-x^2}\cdot y' = 3$ | $y' = \frac{3}{\sqrt{9-x^2}}$ | $y = 3\int \frac{1}{\sqrt{9-x^2}}\,dx$ | $a = 3$, $u = x$ | $du = dx$

$y = 3\int \frac{1}{\sqrt{a^2-u^2}}\,du$ | $y = 3\cdot\arcsin\left(\frac{x}{3}\right)+C$ | $y = 3\arcsin\left(\frac{x}{3}\right)+C$ | A