

BC Retake 5.1 - 5.3

1. Solve for  $x$ :  $\ln(5x - 1) - \ln x = 3$ .

2. Find  $dy/dx$  for  $y = \ln(5 - x)^6$ .

[A]  $6(5-x)^5$     [B]  $\frac{6}{x-5}$     [C]  $\frac{1}{(5-x)^6}$     [D]  $-6(5-x)^5$     [E] None of these

3. Find  $\frac{dy}{dx}$  if  $y = \ln \frac{\sqrt{x}}{5-x}$ .

4. Find the derivative:  $f(x) = \ln \frac{x^2 \sqrt{4x+1}}{(x^3+5)^3}$ .

[A]  $\frac{2}{x} + \frac{1}{2(4x+1)} - \frac{3}{x^3+5}$     [B]  $\frac{2}{x} - \frac{2}{4x+1} - \frac{9x^2}{x^3+5}$     [C]  $\frac{2}{x} + \frac{2}{4x+1} - \frac{9x^2}{x^3+5}$

[D]  $\frac{x}{9x^2(x^3+5)^2 \sqrt{4x+1}}$     [E] None of these

5. Differentiate:  $y = \ln(\ln \tan x)$ .

6. Find  $y'$  if  $\ln xy = x + y$ .

[A]  $\frac{xy-y}{x-xy}$     [B]  $-\frac{y}{x}$     [C]  $\frac{xy}{1-xy}$     [D]  $e^{x+y}$     [E] None of these

7. Use logarithmic differentiation to find  $\frac{dy}{dx}$ :  $y = \frac{(x+2)\sqrt{1-x^2}}{4x^3}$ .

[A]  $\frac{1}{x+2} - \frac{2x}{1-x^2} - \frac{3}{x}$     [B]  $\frac{-x}{12x^2 \sqrt{1-x^2}}$     [C]  $\frac{1}{x+2} - \frac{x}{1-x^2} - \frac{3}{x}$

[D]  $\frac{(x+2)\sqrt{1-x^2}}{4x^3} \left[ \frac{1}{x+2} - \frac{x}{1-x^2} - \frac{3}{x} \right]$     [E] None of these

8. Evaluate the integral:  $\int_e^{4e} \frac{1}{x} dx$ .

9. Find an equation for the tangent line to the graph of  $f(x) = \ln(x^2 - 1)$  at the point where  $x = 2$ .

[A]  $4x - 3y = 8 - \ln 27$

[B]  $4x - 3y = -1$

[C]  $4x - y = 8 - \ln 3$

[D]  $4x - 3y = 8$

[E] None of these

10. Evaluate the integral:  $\int_2^{e+1} \frac{1}{x-1} dx$ .

11. Evaluate the definite integral:  $\int_1^{\sqrt{e}} \frac{2x}{x^2} dx$ .

12. Evaluate the integral:  $\int \frac{3x^2 + 3x + 3}{x^2 + 1} dx$ .

[A]  $3x + \frac{3}{2} \ln(x^2 + 1) + C$

[B]  $3 + \frac{3}{2} \ln(x^2 + 1) + C$

[C]  $3x + 3 \ln(x^2 + 1) + C$

[D]  $3 + 3 \ln(x^2 + 1) + C$

[E] None of these

13. Evaluate the integral:  $\int \sec 2x dx$ .

14. Evaluate the integral:  $\int x \cot x^2 dx$ .

[A]  $x \cot x^2 \csc x^2 + C$

[B]  $\frac{1}{2} x^2 \sec^2 x^2 + C$

[C]  $\frac{1}{4} x^2 \ln|\sin x^2| + C$

[D]  $\frac{1}{2} \ln|\sin x^2| + C$

[E] None of these

15. Given  $f(x) = \frac{2x+1}{3}$ , find  $f^{-1}(x)$ .

16. Evaluate the integral:  $\int \frac{\ln \sqrt{x}}{x} dx$ .

17. Find  $(f^{-1})'(7)$  for  $f(x) = 5 - 2x^3$

18. Find  $(f^{-1})'(2)$  for  $f(x) = \frac{x+3}{x+1}$

$z = \frac{x+3}{x+1}$      $2x+2 = x+3$      $f'(x) = \frac{(x+1)(1) - (x+3)(1)}{(x+1)^2}$      $f'(x) = \frac{x+1-x-3}{(x+1)^2}$      $f'(x) = \frac{-2}{(x+1)^2}$      $(f^{-1})(2) = \frac{1}{f'(f^{-1}(2))}$   
 $x = 1$      $f^{-1}(2) = 1$      $f'(1) = \frac{-2}{4} = -\frac{1}{2}$      $\frac{1}{-\frac{1}{2}} = -2$      $\frac{1}{f'(1)} = -2$

Calculus BC 5.1-5.3 Retake Solutions

①  $\ln(5x-1) - \ln x = 3$      $\ln\left(\frac{5x-1}{x}\right) = 3$      $\frac{5x-1}{x} = \frac{e^3}{1}$     Cross MULTIPLY

$x e^3 = 5x - 1$      $x(e^3 - 5) = -1$      $x = \frac{-1}{e^3 - 5}$   
 $x e^3 - 5x = -1$

②  $y = \ln(5-x)^6$      $y' = 6 \cdot \left(\frac{1}{5-x}\right) \cdot (-1)$      $y' = \frac{-6}{5-x}$      $y' = \frac{6}{x-5}$     B  
 $= 6 \ln(5-x)$

③  $y = \ln\left(\frac{\sqrt{x}}{5-x}\right)$      $y = \ln(x^{1/2}) - \ln(5-x)$      $y' = \frac{1}{x^{1/2}} \cdot \left(\frac{1}{2} x^{-1/2}\right) - \frac{1}{5-x} \cdot (-1)$

$y' = \frac{1}{x^{1/2}} \cdot \frac{1}{2x^{1/2}} + \frac{1}{5-x}$      $y' = \frac{1}{2x} + \frac{1}{5-x}$

④  $f(x) = \ln\left[\frac{x^2 \cdot \sqrt{4x+1}}{(x^3+5)^3}\right]$      $f(x) = \ln x^2 + \ln(4x+1)^{1/2} - \ln(x^3+5)^3$   
 $f(x) = 2 \ln x + \frac{1}{2} \ln(4x+1) - 3 \ln(x^3+5)$

$f'(x) = \frac{2}{x} + \frac{1}{2} \left(\frac{1}{4x+1}\right) \left(\frac{2}{1}\right) - 3 \left(\frac{1}{x^3+5}\right) \left(3x^2\right)$      $f'(x) = \frac{2}{x} + \frac{1}{4x+1} - \frac{9x^2}{x^3+5}$     C

⑤  $y = \ln(\ln(\tan x))$     pssx     $y' = \frac{1}{\ln(\tan x)} \cdot \frac{1}{\tan x} \cdot \sec^2 x$      $y' = \frac{\sec^2 x}{\tan x \cdot \ln(\tan x)}$

⑥  $\ln(xy) = x+y$      $\frac{d}{dx} [\ln(xy)] = \frac{d}{dx} [x+y]$      $1 \cdot \left[ (x) \left(\frac{dy}{dx}\right) + (y) (1) \right] = 1 + \left(\frac{dy}{dx}\right)$   
 $\frac{1}{xy}$

$x \left(\frac{dy}{dx}\right) + y = 1 + \left(\frac{dy}{dx}\right)$      $x \left(\frac{dy}{dx}\right) + y = xy \left(1 + \frac{dy}{dx}\right)$   
 $\frac{x \left(\frac{dy}{dx}\right) + y}{xy}$      $x \left(\frac{dy}{dx}\right) + y = xy + xy \cdot \frac{dy}{dx}$

$x \left(\frac{dy}{dx}\right) - xy \left(\frac{dy}{dx}\right) = xy - y$      $\frac{dy}{dx} = \frac{xy - y}{x - xy}$     A  
 $\left(\frac{dy}{dx}\right) (x - xy) = xy - y$

⑦  $y = \frac{(x+2)\sqrt{1-x^2}}{4x^3}$      $\ln(y) = \ln\left[\frac{(x+2)(1-x^2)^{1/2}}{4x^3}\right]$      $\ln(y) = \ln(x+2) + \ln(1-x^2)^{1/2} - \ln(4x^3)$

$\ln(y) = \ln(x+2) + \frac{1}{2} \ln(1-x^2) - \ln(4x^3)$      $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x+2} + \frac{1}{2} \left(\frac{1}{1-x^2}\right) (-2x) - \frac{1}{4x^3} (12x^2)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x+2} - \frac{x}{1-x^2} - \frac{3}{x}$      $\frac{dy}{dx} = \left[ \frac{1}{x+2} - \frac{x}{1-x^2} - \frac{3}{x} \right] \cdot (y)$     D  
original Problem



⑧  $\int_{\frac{1}{e}}^{4e} \frac{1}{x} dx$   $\ln x$   $\ln 4e - \ln \frac{1}{e}$   $\ln 4e - 1$

⑨  $f(x) = \ln(x^2 - 1)$   $x_1 = 2$   $f(2) = \ln(3)$   $f'(x) = \frac{2x}{x^2 - 1}$   $f'(2) = \frac{2(2)}{4 - 1} = \frac{4}{3}$   
 where  $x = 2$   $y_1 = \ln 3$   $f'(x) = \frac{1}{x^2 - 1} \cdot 2x$   $4 - 1$   $3$   
 $m = \frac{4}{3}$

$y - y_1 = m(x - x_1)$   $3y - 3\ln 3 = 4(x - 2)$   $8 - \ln 27 = 4x - 3y$  A  
 $y - \ln 3 = \frac{4}{3}(x - 2)$   $3y - \ln 3^3 = 4x - 8$   $4x - 3y = 8 - \ln y$   
 $3y - \ln 27 = 4x - 8$

⑩  $\int_2^{e+1} \frac{1}{x-1} dx$   $u = x - 1$   $\int_2^{e+1} \frac{1}{u} du$   $\ln|x-1|$   $F(e+1) - F(2)$   
 $du = dx$

$\ln|e+1-1| - \ln|2-1|$   $\ln e - \ln 1$   $1 - 0$   $1$   
 $\ln|e| - \ln|1|$

⑪  $\int_1^{\sqrt{e}} \frac{2x}{x^2} dx$   $\int_1^{\sqrt{e}} \frac{2}{x} dx$   $2 \int_1^{\sqrt{e}} \frac{1}{x} dx$   $2 \cdot \ln|x|$   $F(\sqrt{e}) - F(1)$

$2 \cdot \ln e^{1/2} - 2 \cdot \ln 1$   $2 [\ln e^{1/2} - \ln(1)]$   $2 [\frac{1}{2} - 0]$   $1$

⑫  $\int \frac{3x^2 + 3x + 3}{x^2 + 1} dx$   $\int \frac{3x^2 + 3}{x^2 + 1} dx + \int \frac{3x}{x^2 + 1} dx$   $3 \int \frac{x^2 + 1}{x^2 + 1} dx + 3 \int \frac{x}{x^2 + 1} dx$

$3 \int 1 dx + 3 \int \frac{2x}{x^2 + 1} dx$   $u = x^2 + 1$   $3x + \frac{3}{2} \int \frac{1}{u} du$   $3x + \frac{3 \cdot \ln(x^2 + 1)}{2} + c$  A  
 $du = 2x dx$

These two, you just have to know the formula. PSST & MGS&Ct won't work!!

⑬  $\frac{1}{2} \int 2 \sec(2x) dx$   $u = 2x$   $\frac{1}{2} \int \sec(u) du$   $\frac{1}{2} \cdot \ln|\sec(2x) + \tan(2x)| + c$   
 $du = 2 dx$

⑭  $\frac{1}{2} \int 2x \cot(x^2) dx$   $u = x^2$   $\frac{1}{2} \int \cot(u) du$   $\frac{1}{2} \cdot \ln|\sin(x^2)| + c$  D  
 $du = 2x dx$

⑮  $f(x) = \frac{2x + 1}{3}$   $x = \frac{2y + 1}{3}$   $3x = 2y + 1$   $y = \frac{3x - 1}{2}$   $f^{-1}(x) = \frac{3x - 1}{2}$   
 $3x - 1 = 2y$

⑯  $\int \frac{\ln \sqrt{x}}{x} dx$   $u = \sqrt{x}$   $dx = 2u \cdot du$   $\int \frac{\ln u \cdot 2u du}{u^2}$   $2 \int \frac{\ln u}{u} du$   $v = \ln u$   $dv = \frac{1}{u} du$   
 $x = u^2$

$2 \int v \cdot dv$   $2 \cdot \frac{v^2}{2} + c$   $v^2 + c$   $(\ln u)^2 + c$   $\ln(\sqrt{x})^2 + c$   $\ln x + c$

⑰  $f(x) = 5 - 2x^3$   $7 = 5 - 2x^3$   $2 = -2x^3$   $x = -1$   $f'(-1) = -6$   $(f^{-1})'(-6) = \frac{1}{f'(-1)} = \frac{1}{-6}$   $f'(x) = -6x^2$   $f'(-1) = -6$

6  
49  
7  
34 3

$\frac{1}{-6}$