Calculus BC Chp. 4 Test Study Guide

Evaluate the Integral

1. $\int x^3 (x^4 - 2)^5 \, dx$

2. $\int \cos^2(4x) \sin(4x) \, dx$

3. $\int x\sqrt{x-2} \, dx$

4. $\int \frac{\csc^2 x}{\sqrt{\cot x}} \, dx$

5. $\int 4 \sec x \tan x \, dx$

6. $\int \cos\left(\frac{x}{3}\right) dx$

7. $\int \frac{2}{\sqrt{4x-7}} \, dx$

8. $\int_1^4 \sqrt{x} \, dx$

9. $\int_{\pi/3}^{2\pi/3} \csc^2 x \, dx$

10. $\int \frac{ax^5 - bx^2}{\sqrt{x}} \, dx$

11. Find the function $y = f(x)$ if $f''(x) = x^2$, $f'(0) = 7$ and $f(0) = 2$.

12. A ball is dropped from a height of 500 feet. Its velocity after t seconds is $v(t) = -32t$ feet/second.

    a) How fast is the ball dropping after 3 seconds?

    b) Determine the position function.

    c) How far has the ball dropped after 3 seconds.

    d) How many seconds will it take for the ball to hit the ground?

13. Find the particular solution of the equation $f'(x) = 4x^{-1/2}$ that satisfies the condition $f(1) = 12$.

(14) Find the limit of $s(n)$ as $n \to \infty$. $s(n) = \sum_{i=1}^{n} \frac{24i}{n^2}$

(15) Find the average value of $f(x) = 2x^2 + 3$ on the interval $[0,2]$.

(16) If $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$, find each of the following:

a) $\int_5^5 f(x)\,dx$ b) $\int_5^0 f(x)\,dx$ c) $\int_0^7 f(x)\,dx$

(17) Use properties of sigma notation and the summation formulas to evaluate the given sum: $\sum_{i=1}^{20} (i-1)^2$

(18) Use the Trapezoidal Rule with 4 equal subintervals to evaluate:

$$\int_0^1 \frac{2}{(x+2)^2}\,dx$$

Calculus B.C. Chp. 4 Test S.G. Solutions

① $\frac{1}{4}\int 4x^3(x^4-2)^5\,dx$ | $u = x^4 - 2$, $du = 4x^3\,dx$ | $\frac{1}{4}\int u^5\,du$ | $\frac{1}{4}\cdot\frac{u^6}{6}+c$

$\frac{1}{4}\cdot\frac{(x^4-2)^6}{6}+c$ | $\boxed{\frac{(x^4-2)^6}{24}+c}$

② $-\frac{1}{4}\int -4\cos^2(4x)\sin(4x)\,dx$ | $u = \cos(4x)$, $du = -4\sin(4x)\,dx$ | $-\frac{1}{4}\int u^2\,du$

$\frac{-1}{4}\int u^2\,du$ | $\frac{-1}{4}\cdot\frac{u^3}{3}+c$ | $\boxed{\frac{-\cos^3(4x)}{12}+c}$

③ $\int x\sqrt{x-2}\,dx$ | $u = x-2$, $x = u+2$ | $du = dx$ | $\int (u+2)\sqrt{u}\,du$ | $\int \left(u^{3/2}+2u^{1/2}\right)du$

$\frac{2u^{5/2}}{5}+\frac{2\cdot 2u^{3/2}}{3}+c$ | $\boxed{\frac{2}{5}(x-2)^{5/2}+\frac{4}{3}(x-2)^{3/2}+c}$

④ $-\int \frac{-\csc^2 x}{\sqrt{\cot x}}\,dx$ | MCsCsC/ | $u = \cot x$, $du = -\csc^2 x\,dx$ | $-\int \frac{1}{u^{1/2}}\,du$ | $-\int u^{-1/2}\,du$

$-\frac{2u^{1/2}}{1}+c$ | $\boxed{-2(\cot x)^{1/2}+c}$

⑤ $\int 4\sec x\tan x\,dx$ | PS8X | $4\int \sec x\tan x\,dx$ | $\boxed{4\sec x + c}$

⑥ $3\int \frac{1}{3}\cos\left(\frac{x}{3}\right)dx$ | $u = x/3$, $du = \frac{1}{3}\,dx$ | $3\int \cos(u)\,du$ | $\boxed{3\sin(x/3)+c}$

⑦ $\frac{1}{2}\int 2.\frac{2}{\sqrt{4x-7}}\,dx$ | $u = 4x-7$, $du = 4\,dx$ | $\frac{1}{2}\int \frac{1}{u^{1/2}}\,du$ | $\frac{1}{2}\int u^{-1/2}\,du$

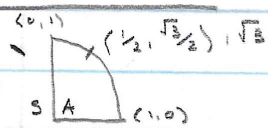
$\frac{1}{2}\cdot\frac{2(u)^{1/2}}{1}+c$ | $\boxed{(4x-7)^{1/2}+c}$

⑧ $\int_1^4 x^{1/2}\,dx$ | $\left.\frac{2x^{3/2}}{3}\right|_1^4$ | $F(4)-F(1)$ | $\frac{2}{3}(4)^{3/2}-\frac{2}{3}$ | $\frac{2}{3}\cdot 8-\frac{2}{3}$ | $\boxed{\frac{14}{3}}$

(9)

$$\int_{\pi/3}^{2\pi/3} \csc^2 x \, dx \qquad \text{M}\cancel{\text{csc}}\text{CT} \qquad -\cot x \Big|_{\pi/3}^{2\pi/3} \qquad F(2\pi/3) - F(\pi/3)$$

$$-\left[\cot(2\pi/3) - \cot(\pi/3)\right] \qquad -\left[\frac{1}{\tan(2\pi/3)} - \frac{1}{\tan(\pi/3)}\right]$$



$$-\left[\frac{1}{-\sqrt{3}} - \frac{1}{\sqrt{3}}\right] \qquad -\left[\frac{\sqrt{3}}{-3} - \frac{\sqrt{3}}{3}\right] \qquad -\left[\frac{-\sqrt{3}}{3} - \frac{\sqrt{3}}{3}\right] \qquad \boxed{\frac{2\sqrt{3}}{3}}$$

(10)

$$\int \frac{ax^5 - bx^2}{\sqrt{x}} \, dx \qquad \int \frac{ax^5}{x^{1/2}} \, dx - \int \frac{bx^2}{x^{1/2}} \, dx \qquad \int ax^{9/2} \, dx - \int bx^{3/2} \, dx$$

$$\frac{2}{11} ax^{11/2} - \frac{2}{5} bx^{5/2} + C \qquad \boxed{\frac{2}{11} ax^{11/2} - \frac{2}{5} bx^{5/2} + C}$$

(11)

$$f''(x) = x^2 \qquad f'(x) = \frac{x^3}{3} + C \qquad f'(0) = 7 = \frac{(0)^3}{3} + C \qquad 7 = 0 + C, \; C = 7$$

$$f'(x) = \frac{x^3}{3} + 7 \qquad f(x) = \frac{x^4}{12} + 7x + C \qquad f(0) = 2 = \frac{(0)^4}{12} + 7(0) + C$$

$$2 = C \qquad \boxed{f(x) = \frac{x^4}{12} + 7x + 2}$$

(12)

a) $v(t) = -32t \qquad v(3) = -32(3) = \boxed{-96 \ \frac{\text{feet}}{\text{second}}}$

b) $s(t) = \frac{-32t^2}{2} + C \qquad s(0) = 500 = -16(0)^2 + C \Rightarrow C = 500 \qquad \boxed{s(t) = -16t^2 + 500}$

c) $s(3) = -16(3)^2 + 500 = -16(9) + 500 \qquad \begin{array}{r} \overset{5}{16} \\ \cdot 9 \\ \hline 144 \end{array} \qquad = -144 + 500 = 356$ feet

Note: At $t = 3$ seconds, this says the ball is 356 feet above the ground. But we were asked how far the ball dropped after 3 seconds, so.....

Starting height = 500 feet

Height at $t=3$ = 356 feet

Ball has dropped: $500 - 356 = \boxed{144 \text{ feet}}$

d) $s(t) = 0 = -16t^2 + 500 \qquad 16t^2 = 500 \qquad t^2 = \frac{500}{16} \qquad t = \pm\frac{\sqrt{100 \cdot 5}}{\sqrt{16}} \qquad t = \frac{10\sqrt{5}}{4} = \boxed{\frac{5\sqrt{5}}{2} \text{ seconds}}$

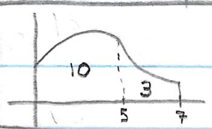
(13) $f'(x) = 4x^{-1/2}$ | $f(x) = 4\cdot\frac{2x^{1/2}}{1} + c$ | $f(1) = 12 = 8(1) + c$, $c = 4$ | $\boxed{f(x) = 8\sqrt{x} + 4}$

(14) $s(n) = \sum_{i=1}^{n} \frac{24i}{n^2}$ | $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{24i}{n^2}$ | $\lim_{n\to\infty} \frac{24}{n^2} \sum_{i=1}^{n} i$ | $\lim_{n\to\infty} \frac{24}{n^2}\left(\frac{n(n+1)}{2}\right)$

$\lim_{n\to\infty} 12\left(\frac{n^2+n}{n^2}\right)$ | $12 \lim_{n\to\infty}\left(1+\frac{1}{n}\right)$ | $12\left[1+0\right]$ | $12(1)$ | $\boxed{12}$

(15) $\frac{1}{b-a}\int_a^b f(x)\,dx$ | $\frac{1}{2-0}\int_0^2 (2x^2+3)\,dx$ | $\frac{1}{2}\left[\frac{2x^3}{3} + 3x\right]_0^2$

$F(2) - F(0)$ | $\frac{1}{2}\left[\left(\frac{16}{3}+6\right) - 0\right]$ | $\frac{1}{2}\left[\frac{16}{3} + \frac{18}{3}\right]$ | $\frac{1}{2}\left[\frac{34}{3}\right]$ | $\boxed{\frac{17}{3}}$

(16) 

a) $\int_5^5 f(x)\,dx = \boxed{0}$ | b) $\int_5^0 f(x)\,dx = \boxed{-10}$ | c) $\int_0^7 f(x)\,dx = \boxed{13}$

(17) $\sum_{i=1}^{20} (i-1)^2$ | $\sum_{i=1}^{20}(i^2 - 2i + 1)$ | $\sum_{i=1}^{20} i^2 - 2\sum_{i=1}^{20} i + \sum_{i=1}^{20} 1$

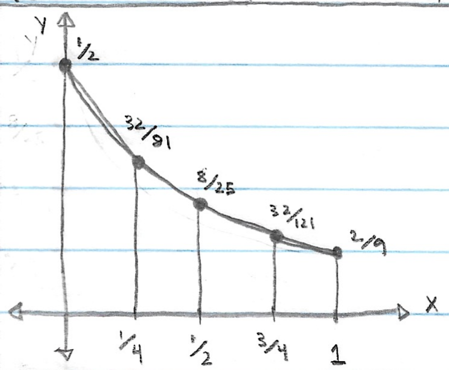
70 × 41 = 70 + 2800 = 2870
2870 − 400 = 2470

$\left[\frac{\cancel{20}^{10}(\cancel{21}^{7})(41)}{\cancel{6}_{3}}\right] - \cancel{2}\left[\frac{20(21)}{\cancel{2}}\right] + 1(20)$ | $\left[70(41) - 21(20) + 1(20)\right]$

$\left[70(41) - 20(20)\right] = \boxed{2,470}$

(18) $\int_0^1 \frac{2}{(x+2)^2}\,dx$ | $f(0) = \frac{2}{4} = \frac{1}{2}$ | $f(1/4) = \frac{2}{\left(\frac{1}{4}+\frac{8}{4}\right)^2} = \frac{2}{(9/4)^2} = \frac{2/1}{81/16} = \frac{2}{1}\cdot\frac{16}{81} = \frac{32}{81}$

$A_T = \frac{1}{2}(b_1 + b_2)h$



$f(1/2) = \frac{2}{\left(\frac{1}{2}+\frac{4}{2}\right)^2} = \frac{2}{(5/2)^2} = \frac{2/1}{25/4} = \frac{2}{1}\cdot\frac{4}{25} = \frac{8}{25}$

$f(3/4) = \frac{2}{\left(\frac{3}{4}+\frac{8}{4}\right)^2} = \frac{2}{(11/4)^2} = \frac{2/1}{121/16} = \frac{2}{1}\cdot\frac{16}{121} = \frac{32}{121}$

$f(1) = \frac{2}{(1+2)^2} = \frac{2}{(3)^2} = \frac{2}{9}$

$A \approx \frac{1}{2}\left(\frac{1}{2} + \frac{32}{81}\right)\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{32}{81} + \frac{8}{25}\right)\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{8}{25} + \frac{32}{121}\right)\left(\frac{1}{4}\right) + \frac{1}{2}\left(\frac{32}{121} + \frac{2}{9}\right)\left(\frac{1}{4}\right)$

$A \approx \frac{1}{8}\left(\frac{1}{2} + \frac{64}{81} + \frac{16}{25} + \frac{64}{121} + \frac{2}{9}\right)$ | $\boxed{A \approx 0.335}$