

1. $y = x^4 - 4x^3 + 16x$	$y' \quad - \quad + \quad +$	$y'' \quad + \quad - \quad +$
$y' = 4(x+1)(x-2)^2$	$-1 \quad 2$	$0 \quad 2$
$y'' = 12x(x-2)$		

Rel. Extrema: $f(-1) = 1 + 4 - 16 = -11$

Rel. Min @ $(-1, -11)$

Inf. Points: $f(0) = 0$

Inf. Pts. @ $(0, 0)$ and $(2, 16)$

$f(2) = 16 - 32 + 32 = 16$

x -int $0 = x^4 - 4x^3 + 16x$

$x \approx -1.679$

you'll have a nice intercept on the test

a. Domain: $(-\infty, \infty)$

b. Range: $[-11, \infty)$

c. Asymptotes: None

d. Rel. Extrema: Rel. Min @ $(-1, -11)$

e. Inc on $(-1, \infty)$, Dec on $(-\infty, -1)$

f. Inf. Pts. @ $(0, 0)$ & $(2, 16)$

g. C.U. on $(-\infty, 0) \cup (2, \infty)$

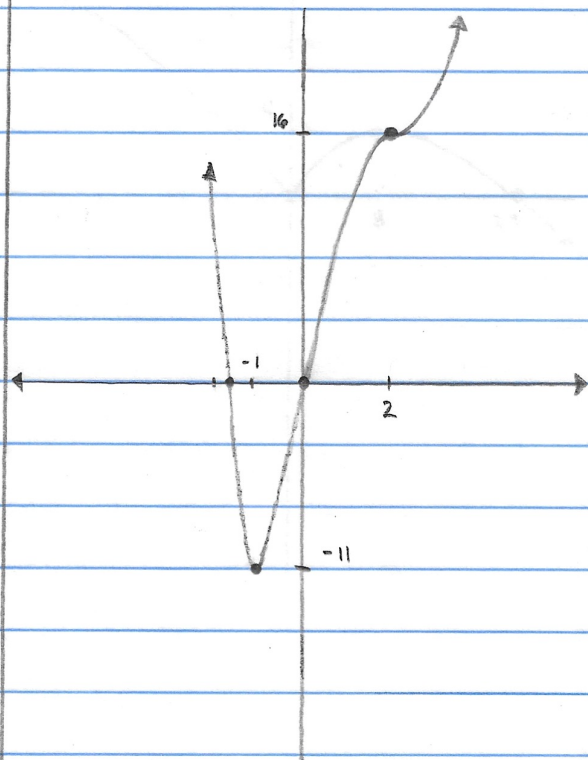
C.D. on $(0, 2)$

h. On Right

i. x -int: $(-1.679, 0)$; y -int: $(0, 0)$

j. Symmetry: None

k. Critical #'s: $x = -1, 2$



$$2. \quad y = 3x^{2/3} - x \quad y' = \frac{2 - x^{1/3}}{x^{1/3}} \quad y'' = \frac{-2}{3x^{4/3}}$$

critical #'s: $2 - x^{1/3} = 0$	$x = (2)^3$	y'	$0 = x^{2/3}(3 - x^{1/3})$
$2 = x^{1/3}$	$= 8$	$\frac{-}{0} \quad \frac{+}{8} \quad \frac{-}{}$	$x = 0, 27$

P.I.P.s: $x = 0$	y''	Rel. Extrema: $f(8) = 3(4) - 8 = 4$
	$\frac{-}{0} \quad \frac{-}{}$	Rel. Min @ $(0, 0)$; Rel. Max @ $(8, 4)$

a. Domain: $(-\infty, \infty)$

b. Range: $(-\infty, \infty)$

c. Asymptotes: None

d. Rel. Extrema: Rel. Min @ $(0, 0)$

Rel. Max @ $(8, 4)$

e. Inc. on $(0, 8)$, Dec. on $(-\infty, 0) \cup (8, \infty)$

f. Inf. Points: None

g. C.D. on $(-\infty, 0) \cup (0, \infty)$

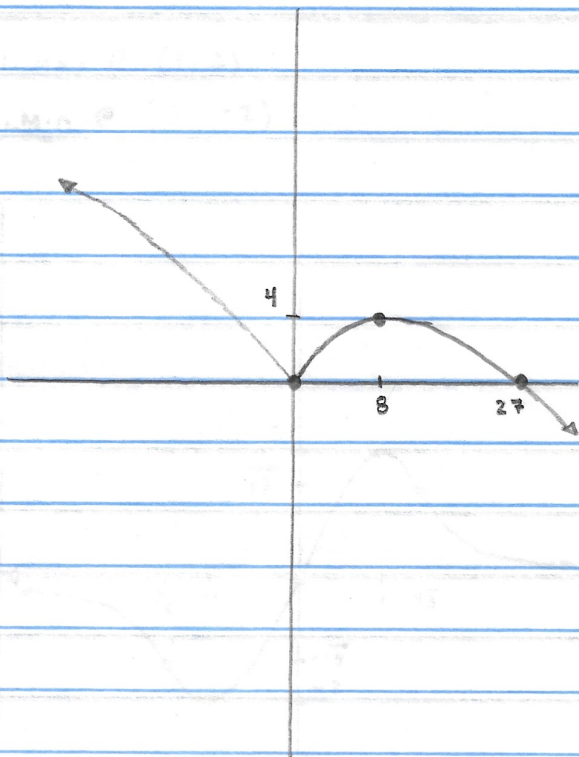
h. on Right

i. x-int: $(0, 0)$, $(27, 0)$

y-int: $(0, 0)$

j. Symmetry: None

k. critical #'s: $x = 0, 8$



3. $f(x) = \frac{4x}{x^2 + 1}$ $f'(x) = \frac{-4(x^2 - 1)}{(x^2 + 1)^2}$ $f''(x) = \frac{8x(x^2 - 3)}{(x^2 + 1)^3}$

y'	-	+	-	y''	-	+	-	+	$f''(1) = +(-)$	$f''(2) = +(+)$
	-1	1			$-\sqrt{3}$	0	$\sqrt{3}$		+	$f''(-2) = -(+)$

Inflection pts : $f(-\sqrt{3}) = \frac{-4\sqrt{3}}{4}$	$f(0) = 0$	$f(\sqrt{3}) = \frac{4\sqrt{3}}{4}$
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Inflection Points at : $(0, 0)$, $(-\sqrt{3}, -\sqrt{3})$, and $(\sqrt{3}, \sqrt{3})$

Rel. Extrema : $f(-1) = \frac{-4}{2}$ = -2	$f(1) = \frac{4}{2}$ = 2	Rel. Max @ $(1, 2)$ Rel. Min @ $(-1, -2)$
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a. Domain : $(-\infty, \infty)$

b. Range : $[-2, 2]$

c. Asymptotes : $y = 0$

d. Rel. Min @ $(-1, -2)$, Rel. Max @ $(1, 2)$

e. Inc. on $(-1, 1)$ & Dec. on $(-\infty, -1) \cup (1, \infty)$

f. Inf. pts @ $(0, 0)$, $(-\sqrt{3}, -\sqrt{3})$, and $(\sqrt{3}, \sqrt{3})$

g. C.U. on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$ &

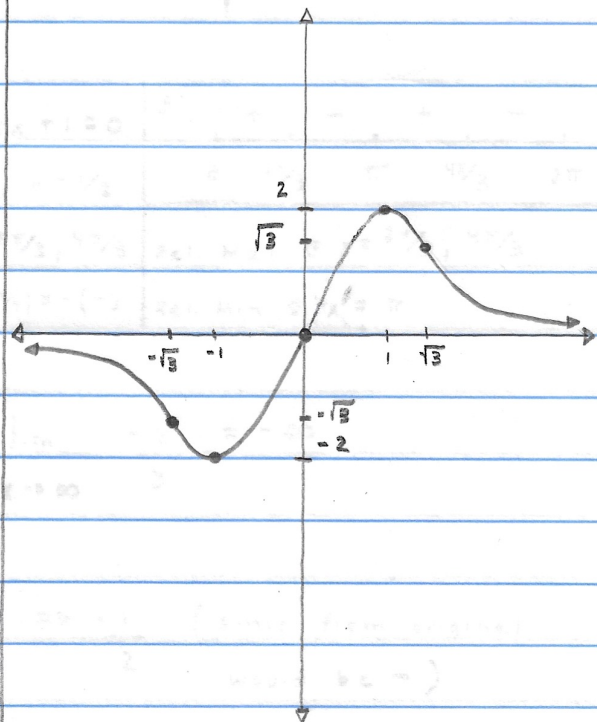
C.D. on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

h. on Right

i. x & y-int : $(0, 0)$

j. symmetry : origin

k. critical #'s : $x = -1, 1$



1.	$f(x) = 3x^4 - 4x^3$	x	y		Abs. Max @
	$f'(x) = 12x^3 - 12x^2$	-1	7	$f(-1) = 3 - 4(-1) = 3 + 4 = 7$	$f(2) = 16$
	$= 12x^2(x-1)$	0	0	$f(0) = 0$	
	c.N.: $x = 0, 1$	1	-1	$f(1) = 3 - 4 = -1$	Abs. Min @
		2	16	$f(2) = 3(16) - 4(8) = 48 - 32 = 16$	$f(1) = -1$

2. since f is not continuous on $[-3, 0]$, MVT does not apply.
 $(f(-1) = \emptyset, \text{ and } -1 \in (-3, 0))$

3.	$f(x) = x^3/6 - 8x$	$0 = x^2/2 - 8$	f'	$+$ $-$ $+$	Rel. Max @ $x = -4$
	$f'(x) = 3x^2/6 - 8$	$8 = x^2/2$		-4 4	Rel. Min @ $x = 4$
	$= x^2/2 - 8$	$16 = x^2 \Rightarrow x = \pm 4$			

4.	$f(x) = \sin^2 x + \sin x$	$\sin x = 0 \ \& \ 2\cos x + 1 = 0$	f'	$+$ $-$ $+$ $-$	
	$= (\sin x)^2 + \sin x$	$x = 0, \pi$		0 $2\pi/3$ π $4\pi/3$ 2π	
	$f'(x) = 2\sin x \cos x + \sin x$	$\cos x = -1/2$			Rel. Max @ $x = 2\pi/3, 4\pi/3$
	$= \sin x (2\cos x + 1)$	$x = 2\pi/3, 4\pi/3$			Rel. Min @ $x = \pi$
		$f'(3\pi/4) = +(-)$; $f'(5\pi/4) = -(-)$			

$$5. \lim_{x \rightarrow \infty} \frac{x^2 - x^3 + 4}{3x^2 + 4x - 1} = \lim_{x \rightarrow \infty} \frac{-x^3}{3x^2} = \lim_{x \rightarrow \infty} \frac{-x}{3} = -\infty$$

$$6. \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{4x^2 - 1}} = \lim_{x \rightarrow -\infty} \frac{1}{\sqrt{4 - 1/x^2}} = \frac{1}{2} \Rightarrow \frac{-1}{2} \quad (\text{since from original would be } -)$$

$$7. \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2 + 9x}) \cdot (x + \sqrt{x^2 + 9x})}{(x + \sqrt{x^2 + 9x})} \Big| \lim_{x \rightarrow \infty} \frac{x^2 - x^2 - 9x}{x + \sqrt{x^2 + 9x}} = \frac{1/x}{1/x} \quad \text{As } x \rightarrow \infty, \frac{1}{x} = \frac{1}{\sqrt{x^2}}$$

$$\lim_{x \rightarrow \infty} \frac{-9}{1 + \sqrt{x^2 + 9x}/\sqrt{x^2}} = \lim_{x \rightarrow \infty} \frac{-9}{1 + \sqrt{1 + 9/x}} = \lim_{x \rightarrow \infty} \frac{-9}{1 + 1} = \frac{-9}{2}$$