Calculus BC CHAPTER 2 STUDY GUIDE

INSTRUCTIONS: Find the derivative

① $f(x) = \dfrac{x - 3x^2}{\sqrt[3]{x}}$

② $f(x) = \dfrac{x^3 - 2x}{x^3}$

③ $f(\theta) = \theta \cdot \sec\theta$

④ $f(x) = \sqrt{\dfrac{x}{x-2}}$

⑤ $y = 2\sin x - 3\cos x + x^2$

⑥ $y = \dfrac{3x}{(2-x^2)^2}$

⑦ $y = \sqrt{x}\,(2x-3)$

⑧ $f(x) = \dfrac{x^2 + 2}{x^2 - 2}$

⑨ $s(t) = \sec(t/3)$

⑩ $f(t) = \dfrac{t}{1 + \sin(t)}$

⑪ $y = \sqrt[3]{x^2 + x}$

⑫ $y = (3x^2 + 2x - 4)^7$

⑬ $f(x) = \dfrac{1}{x^3}$

⑭ $y = \tan(x^3 - \pi/2)$

⑮ $f(x) = x^3 + 2\sec x$

⑯ $f(x) = (3x^3 - 2)^8$

⑰ $f(x) = 3(2-x)^3 + 5(1-x) + 20$

⑱ A particle moves on the curve $y = (t^3 + 2)^2$. Find the acceleration when $t = 3$ seconds.

(19) The position function for an object is given by $s(t) = 4t^3 + 120t$, where s is measured in feet, and t is measured in seconds. Find the velocity of the object when $t = 3$ seconds. Indicate units

(20) The volume of a particular tank is $V = \pi r^3$ where r measures the radius of the tank. If water is flowing in at a rate of 10 $ft.^3/min$, find the rate at which the radius is changing when the radius is 4 feet.

(21) The radius of a circle is increasing at a rate of 7 inches per minute. At what rate is the area increasing when the radius is 10 inches?

(22) Find dy/dx given $4xy + 2x^2 - 3y^3 = 0$.

BC Calc CHP. 2 S.G. SOLUTIONS

① $f(x) = (x - 3x^2) \div x^{1/3}$

$= x^{2/3} - 3x^{5/3}$

$f'(x) = \frac{2}{3} \cdot x^{-1/3} - 5x^{2/3}$

② $f(x) = (x^3 - 2x) \div x^3$

$= 1 - 2x^{-2}$

$f'(x) = 4x^{-3}$

PSST ③ $f(\theta) = \theta \cdot \sec\theta$

$f'(\theta) = \theta \cdot \sec\theta \cdot \tan\theta + \sec\theta$

$= \sec\theta(\theta \cdot \tan\theta + 1)$

④ $f(x) = \left(\frac{x}{x-2}\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(\frac{x}{x-2}\right)^{-1/2}\left(\frac{(x-2)(1)-(x)(1)}{(x-2)^2}\right)$

$= \frac{1}{2}\left(\frac{x}{x-2}\right)^{-1/2}\left(\frac{-2}{(x-2)^2}\right)$

⑤ $y = 2\sin x - 3\cos x + x^2$

$y' = 2\cos x + 3\sin x + 2x$

⑥ $y = 3x \div (2 - x^2)^2$

$y' = \frac{(2-x^2)^2(3) - (3x)(2(2-x^2)(-2x))}{(2-x^2)^4}$

$y' = \frac{3(2-x^2)^2 + 12x^2(2-x^2)}{(2-x^2)^4}$

⑦ $y = x^{1/2}(2x - 3)$

$y' = (x^{1/2})(2) + (2x-3)(\frac{1}{2} \cdot x^{-1/2})$

⑧ $f(x) = (x^2 + 2) / (x^2 - 2)$

$f'(x) = \frac{(x^2-2)(2x) - (x^2+2)(2x)}{(x^2-2)^2}$

⑨ $s(t) = \sec(t/3)$; PSST

$s'(t) = \frac{1}{3} \cdot \sec(t/3) \cdot \tan(t/3)$

⑩ $f(t) = t / (1 + \sin(t))$

$f'(t) = \frac{(1+\sin t)(1) - (t)(\cos t)}{(1+\sin t)^2}$

⑪ $y = (x^2 + x)^{1/3}$

$y' = \frac{1}{3}(x^2 + x)^{-2/3}(2x + 1)$

⑫ $y = (3x^2 + 2x - 4)^7$

$y' = 7(3x^2 + 2x - 4)^6(6x + 2)$

⑬ $f(x) = x^{-3}$; $f'(x) = -3x^{-4}$

⑭ $y = \tan(x^3 - \pi/2)$; PSST

$y' = 3x^2 \cdot \sec^2(x^3 - \pi/2)$

⑮ $f(x) = x^3 + 2\sec x$; PSST

$f'(x) = 3x^2 + 2\sec x \cdot \tan x$

(16) $f(x) = (3x^3 - 2)^8$

$f'(x) = 8(3x^3 - 2)^7 (9x^2)$

$= 72x^2 (3x^3 - 2)^7$

(17) $f(x) = 3(2-x)^3 + 5(1-x) + 20$

$f'(x) = 9(2-x)^2(-1) - 5 + 0$

$= -9(2-x)^2 - 5$

(18) $s(t) = (t^3 + 2)^2$

$v(t) = 2(t^3 + 2)(3t^2)$

$= 6t^2 (t^3 + 2)$

$a(t) = (6t^2)(3t^2) + (t^3 + 2)(12t)$

$a(3) = 54(27) + (29)(36)$

$= 1458 + 1044$

$= 2502$

(19) $s(t) = 4t^3 + 120t$

$v(t) = 12t^2 + 120$

$v(3) = 12(9) + 120$

$= 108 + 120 = 228 \text{ ft./s}$

(20) K : $dV/dt = 10 \text{ ft.}^3/\text{min}$

W : dr/dt when $r = 4$ ft.

E : $V = \pi r^3$

D : $dV/dt = 3\pi r^2 \cdot dr/dt$

$dV/dt = 3\pi r^2 \cdot dr/dt$

$10 \text{ ft.}^3/\text{min} = 3\pi (4 \text{ ft.})^2 \cdot dr/dt$

$10 \text{ ft.}^3/\text{min} = 48\pi \text{ ft.}^2 \cdot dr/dt$

$dr/dt = 10 \text{ ft.}^3/\text{min} \cdot 1/48\pi \text{ ft.}^2$

$= 5/24\pi \text{ ft./min}$

(21) K : $dr/dt = 7$

W : dA/dt when $r = 10$ in.

E : $A = \pi r^2$

D : $dA/dt = 2\pi r \cdot dr/dt$

$dA/dt = 2\pi r \cdot dr/dt$

$dA/dt = 2\pi (10 \text{ in}) \cdot (7 \text{ inch/min})$

$dA/dt = 140\pi \text{ inches}^2/\text{min}$

(22) $4xy + 2x^2 - 3y^3 = 0$

$(4x)(dy/dx) + (y)(4) + 4x - 9y^2 \cdot dy/dx = 0$

$dy/dx (4x - 9y^2) = -4x - 4y$

$$\frac{dy}{dx} = \frac{-4x - 4y}{4x - 9y^2}$$