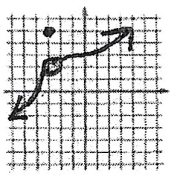
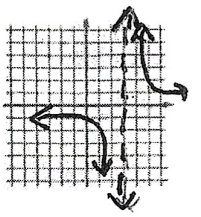
BC (Study Guide)

Calc 2 Chapter 1 Test RETAKE Name_____

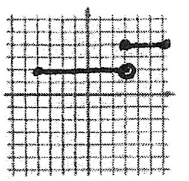
Find the limit or state that it does not exist:



1) $\lim_{x\to -3} f(x)$



2) $\lim_{x\to 3} f(x)$



3) $\lim_{x\to 0} f(x)$

4) $\lim_{x\to 0} \dfrac{\sqrt{9+x}+3}{x}$

5) $\lim_{x\to 1^+} f(x)$ for $f(x)=\begin{pmatrix} 3x-4, x\le 1 \\ 2x^2, x>1 \end{pmatrix}$

6) $\lim_{x\to 2} \dfrac{x-2}{x^3-8}$

7) $\lim_{x\to -1} \dfrac{x^2+5x+2}{x^2+2}$

8) $\lim_{x\to 0} \dfrac{1-\cos x}{x}$

9) $\lim_{x\to 1} \csc\dfrac{\pi x}{6}$

10) $\lim_{\theta\to 0} \dfrac{\sin 6\theta}{\theta}$

11) $\lim_{x\to 0} \dfrac{\frac{1}{x+2}-\frac{1}{2}}{x}$

12) $\lim_{x\to \pi} \cot x$

13) $\lim_{x\to 1} \dfrac{\sin^{-1} x}{\tan^{-1} x - 1}$

14) Find $\lim_{x\to 4} \dfrac{x-4}{|x-4|}$

15) If $\lim_{x\to c} f(x) = \dfrac{3}{5}$ and $\lim_{x\to c} g(x) = \dfrac{2}{3}$, find $\lim_{x\to c} \dfrac{f(x)}{g(x)}$.

16) For $f(x)=\dfrac{2}{(x-3)(5-x)}$, f(x) decreases without bound as x approaches what value from the left?

17) For what value(s) of c will f(x) be continuous for $f(x)=\begin{pmatrix} cx+3, x\le 5 \\ cx^2, x>5 \end{pmatrix}$

18) Determine the intervals on which the function is continuous for $f(x)=\dfrac{3x-1}{6x+12}$.

19) Name the 3 ways a limit fails to exist.

20) Use the Intermediate Value Theorem to explain why the function $x^4 - 3x^2 + 4x$ must have a zero in the interval [-3,-2].

BC CALCULUS: Test 1 Study Guide Solutions

① $\lim_{x \to -3} f(x) = 2$ ② $\lim_{x \to 3} f(x) = \text{DNE}$ ③ $\lim_{x \to 0} f(x) = 2$

④

$\lim_{x \to 0} \frac{\sqrt{9+x} + 3}{x}$	$\frac{\sqrt{9+0} + 3}{0}$	$\frac{3+3}{0}$	$\frac{6}{0}$	DNE

⑤

As $x \to 1^+$, use $f(x) = 2x^2$	$f(1) = 2(1)^2 = 2$

⑥

$\lim_{x \to 2} \frac{x-2}{x^3-8}$	$\frac{0}{0}$ $\Rightarrow$ L.R.A.	$\lim_{x \to 2} \frac{1}{3x^2}$	$\frac{1}{3(2)^2}$	$\frac{1}{12}$

⑦

$\lim_{x \to -1} \frac{x^2 + 5x + 2}{x^2 + 2}$	$\frac{(-1)^2 + 5(-1) + 2}{(-1)^2 + 2}$	$\frac{1 - 5 + 2}{1 + 2}$	$\frac{-2}{3}$

⑧

$\lim_{x \to 0} \frac{1 - \cos x}{x}$	$\frac{0}{0}$ $\Rightarrow$ L.R.A.	$\lim_{x \to 0} \frac{\sin x}{1}$	$\sin(0)$	0

⑨

$\lim_{x \to 1} \csc\left(\frac{\pi x}{6}\right)$	$\csc(\pi/6)$	$\frac{1}{\sin(\pi/6)}$	$\frac{1}{(1/2)}$	$\frac{1}{1} \cdot \frac{2}{1}$	2

⑩

$\lim_{\theta \to 0} \frac{\sin(6\theta)}{\theta}$	$\frac{\sin(0)}{0}$	$\frac{0}{0}$ $\Rightarrow$ L.R.A.	$\lim_{\theta \to 0} \frac{6 \cdot \cos(6\theta)}{1}$	$6 \cdot \cos(0)$	$6(1) = 6$

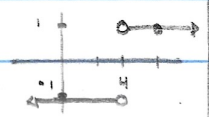
⑪

$\lim_{x \to 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x}$	$\lim_{x \to 0} \frac{\frac{2}{2(x+2)} - \frac{1(x+2)}{2(x+2)}}{x}$	$\lim_{x \to 0} \frac{\frac{2-(x+2)}{2(x+2)}}{x}$

$\lim_{x \to 0} \frac{-x}{2(x+2)} \cdot \frac{1}{x}$	$\lim_{x \to 0} \frac{-1}{2(x+2)}$	$-1/2(2)$	$-\frac{1}{4}$

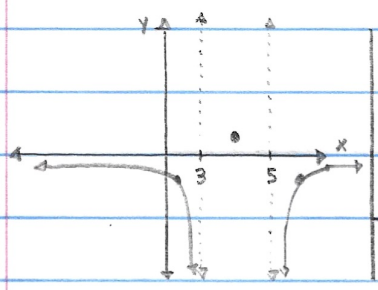
(12) $\lim_{x \to \pi} \cot(x)$ | $\cot(\pi)$ | $\frac{1}{\tan(\pi)}$ | $\frac{1}{0}$ | $\varnothing$ | DNE

(13) $\lim_{x \to 1} \frac{\sin^{-1}(x)}{\tan^{-1}(x) - 1}$ | $\frac{\pi/2}{\pi/4 - 1}$ | $\frac{\pi/2}{\pi/4 - 4/4}$ | $\frac{\pi/2}{\pi - 4/4}$ | $\frac{\pi}{2} \cdot \frac{4^2}{\pi - 4}$ | $\frac{2\pi}{\pi - 4}$

(14) $\lim_{x \to 4} \frac{x - 4}{|x - 4|}$ |  | $f(5) = 1/1$ | DNE

$f(6) = 2/2$

(15) $\lim_{x \to c} \frac{f(x)}{g(x)}$ | $\frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ | $\frac{3/5}{2/3}$ | $\frac{3}{5} \cdot \frac{3}{2}$ | $\frac{9}{10}$

(16) 

$f(4) = \frac{2}{(1)(1)} = 2$ | $f(2) = \frac{2}{(-1)(3)} = -2/3$

$f(6) = \frac{2}{(3)(-1)} = -2/3$ | $f(0) = 2/-15$

$f(7) = \frac{2}{(4)(-2)} = -1/4$

f decreases w/o bound as $x \to 3^-$

(17) $f(x) = \begin{cases} cx + 3, & x \leq 5 \\ cx^2, & x > 5 \end{cases}$ | $cx + 3 = cx^2$ | $3 = 20c$ | $c = \frac{3}{20}$

$5c + 3 = 25c$

(18) $6x + 12 = 0$ | $x = -2$ | f is continuous on $(-\infty, -2) \cup (-2, \infty)$

$6x = -12$

(19) ① one sided limits dont equal ③ V. A.

② Endpoint ④ oscillating

(20) $f(-3) = 81 - 3(9) - 12$ | $f(-2) = 16 - 12 - 8$ | Since f is continuous on $[-3, -2]$,

$= 81 - 39$ | $= 16 - 20$ | $f(-3) = 42 \neq -4 = f(-2)$, & $0 \in (-4, 42)$

$= 42$ | $= -4$ | then there must exist at least one $c \in (-3, -2)$ s.t. $f(c) = 0$ by I.V.T.